類組：物理類　科目：應用數學(2001)　※請在答案卷內作答

1. (10%) If a vector field $\mathbf{v}$ is expressed as $\mathbf{v} = \omega \times \mathbf{r}$, where $\omega = \omega_1\hat{\mathbf{i}} + \omega_2\hat{\mathbf{j}} + \omega_3\hat{\mathbf{k}}$ and $\mathbf{r}$ is the position vector in an orthogonal coordinate system with unit vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$. Determine, showing all the work, the curl of the vector field $\mathbf{v}$.

2. (15%) Let $\mathbf{F} = \dfrac{z\hat{\mathbf{j}} - y\hat{\mathbf{k}}}{y^2+z^2}$. Please answer the following questions:

(a) (5%) Calculate, showing all the work, $\nabla \times \mathbf{F}$.

(b) (10%) Evaluate the integral $\oint \mathbf{F}\cdot d\mathbf{r}$ around any closed loop surrounding the origin.

3. (25%) Let $A = \begin{vmatrix} 1 & -1 & 2 \\ 5 & -5 & 2 \\ 4 & -6 & 4 \end{vmatrix}$.

(a) (8%) Find the eigenvalues of A.

(b) (10%) Find a maximal set of linearly independent eigenvectors of A.

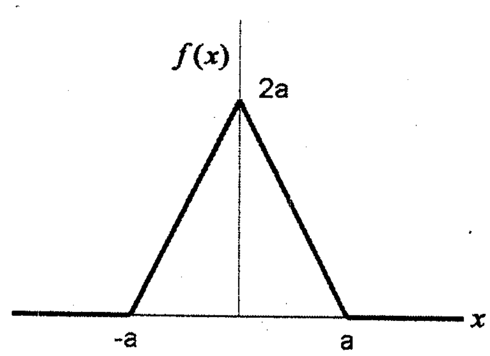
(c) (7%) Is the matrix A diagonalizable (Yes or No)? If yes, find P such that $D = P^{-1}AP$ is diagonal. If not, explain your answer based on the results obtained in (a) and (b).

4. (5%) Determine the first three expansion terms of the Legendre series of a function $f(x)$ given by $f(x) = \begin{cases} 1, & -1 < x < 0 \\ 0, & 0 < x < 1 \end{cases}$.

5. (10%) Evaluate the integral, $J = \int_V e^{-r}\left(\nabla \cdot \dfrac{\hat{r}}{r^2}\right) d\tau$, where V is a sphere of radius R centered at the origin, by the method of integration by part.

6. (10%) Find the general solution of the ordinary differential equation, $(x\ln x)y' + y = \ln x$.

7. (10%) Find the exponential Fourier transform of the following $f(x)$ and write $f(x)$ as a Fourier integral.



8. (15%) (a) (6%) Show that the Laplace transform of $f(t) = te^{3t}$ is $F(s) = \dfrac{1}{(s-3)^2}$. (b) (9%) By using Laplace transform, solve the differential equation, $y'' - 6y' + 9y = te^{3t}$, $y_0 = 0$, $y'_0 = 5$.

參考用