

國 立 清 華 大 學 命 題 紙

96 學年度生醫工程與環境科學系丙組(醫學物理與工程)組碩士班入學考試
科目應用數學 科目代碼 2702 共 1 頁第 1 頁 *請在【答案卷卡】內作答

1. (10%). Find the eigenvalues and eigenvectors of the following matrix

$$A = \begin{bmatrix} 3 & -2 & 2 \\ 6 & -4 & 6 \\ 2 & -1 & 3 \end{bmatrix}.$$

2. (10 %). Solve the initial value problem of the differential equation:

$$x \frac{dy}{dx} - y = x^2 \cos x, \quad y(\pi/2) = \pi.$$

3. (10%). Solve the initial value problem $y'' + 3y' + 2y = \sin 3x$ with $y(0) = 0$ and $y'(0) = 1$.

4. (10%). Find the general solution of the following differential equation

$$x^2 y'' + 3xy' - 8y = 0.$$

5. (10%). Solve the initial value problem $x^2 y'' - 3xy' + 3y = 2x^2 \ln x$ with $y(1) = 0$ and $y'(1) = 0$.

6. (10%). Solve the homogeneous system of differential equations

$$x_1'(t) = x_2 + x_3, \quad x_2'(t) = x_1 + x_3, \quad x_3'(t) = x_2 + x_1.$$

7. (10%). Find Laplace transforms of (a) $t \sin at$, and (b) $te^{at} \cos bt$.

8. (10%). Consider the differential equation $x^2(\ln x - 1)y'' - xy' + y = 0$ show that $y_1(x) = x$ is a solution and find a second linear independent solution.

9. (10%). Find the Fourier series representation of the function $f(x) = x + 1$ in the interval $-1 \leq x \leq 1$.

10. (10%). Find the residues at the poles of the complex function (a) $f(z) = \frac{1}{(z^2 + 1)^2}$, and

(b) $f(z) = \sin\left(\frac{z}{z+1}\right)$.