注意：考試開始鈴響前，不得翻閱試題，
並不得書寫、畫記、作答。

國立清華大學 109 學年度碩士班考試入學試題
系所班組別：核子工程與科學研究所
科 目 代 碼 ：3201
考 試 科 目 ：工程數學

—作答注意事項—

1. 請核對答案卷（卡）上之准考證號、科目名稱是否正確。
2. 作答中如有發現試題印刷不清，得舉手請監試人員處理，但不得要求解釋題意。
3. 考生限在答案卷上標記「由此開始作答」區內作答，且不可書寫姓名、准考證號或與作答無關之其他文字或符號。
4. 答案卷用盡不得要求加頁。
5. 答案卷可用任何書寫工具作答，惟為方便閱卷辨識，請儘量使用藍色或黑色書寫；答案卡限用 2B 鉛筆畫記；如畫記不清（含未依範例畫記）致光學閱讀機無法辨識答案者，其後果一律由考生自行負責。
6. 其他應考規則、違規處理及扣分方式，請自行詳閱准考證明上「**國立清華大學試場規則及違規處理辦法**」，無法因本試題封面作答注意事項中未列明而稱未知悉。

國立清華大學 109 學年度碩士班考試入學試題

系所班組別：核子工程與科學研究所(0532)

考試科目（代碼）：工程數學(3201)

　　*請在【答案卷】作答

1. Solve the differential equations of $y(x)$.

(a) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = -e^{2x}$ (5 %)

(b) $(x+1)^2\frac{dy}{dx} + 3y = 2 - 3xy,$ (5 %)

(c) $(x^3y + 2y)\frac{dy}{dx} = -x^2$; show the particular solution for $y(0) = 2$. (5 %)

2. Use the Laplace transform to solve the problem

$$\frac{dx}{dt} + 2x = f(t),\ \text{where}\ f(t) = \begin{cases} 0, & 0 \le t < 1 \\ 1, & 1 \le t < 2 \\ 0, & t \ge 2 \end{cases}$$

and $x(0) = 0$. You may express $f(t)$ in terms of unit step functions. (10 %)

3. Find the series solution of the following differential equation about $x = 0$.

$$3x\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0\ .$$ (10 %)

You have to express the solution in the form of $y(x) = C_1y_1(x) + C_2y_2(x)$. To save time, you can only show the first three terms of $y_1(x)$ and $y_2(x)$.

4. Consider the matrix

$$M = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & -1 \\ -3 & 1 & 4 \end{bmatrix}.$$

(a) Find the determinant of A and obtain the inverse matrix A^{-1} (7 %).

(b) Estimate the eigenvalues and eigenvectors of A (8 %).

5. (a) Evaluate $\oint_C 2zdx + 3xdy + ydz$, where C is the trace of the cylinder $x^2 + y^2 = 4$ in the plane $y + z = 3$. Orient C counterclockwise as viewed from above.

(5%)

(b) Use divergence theorem to find the outward flux $\iint_S (\mathbf{F}\cdot\mathbf{n})dS$ where $\mathbf{F} = y^2\mathbf{i} + x^2z^2\mathbf{j} + (z-2)^2\mathbf{k}$ and S is the surface of the region bounded by the cylinder $x^2 + y^2 = 25$ and the planes $z = 2,\ z = 6$. (5%)

6. Solve the system of linear differential equations:

$$\mathbf{X}' = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix}\mathbf{X} + \begin{pmatrix} t \\ t+1 \end{pmatrix}, \quad \mathbf{X}(0) = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \qquad (10\%)$$

7. (a) Expand $f(x) = \frac{1}{2}|\sin x|,\ -\pi < x < \pi$, in a Fourier-cosine series. (5%)

(b) Write out the first three nonzero terms in the Fourier-Legendre expansion of

$$f(x) = \begin{cases} 1, & -1 < x < 0 \\ 0, & 0 \le x < 1 \end{cases}. \qquad (5\%)$$

Hints: (1) Fourier-Legendre series: $f(x) = \sum_{n=0}^{\infty} c_n P_n(x)$, where $c_n = \frac{2n+1}{2}\int_{-1}^{1} f(x)P_n(x)dx$. (2) Rodrigues' formula: $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$.

8. Solve the Dirichlet problem: $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0,\ 0 < x < 5,\ 0 < y < 10$ with $u(0,y) = 0,\ u(5,y) = 0,\ u(x,0) = 0,\ u(x,10) = 3$. (10%)

9. (a) Use the residue theorem to evaluate $\oint_C \frac{z}{(z+2)(z^2+4)}dz$, where C is the curve $25x^2 + y^2 = 25$. Orient C counterclockwise. (5%)

(b) Evaluate Cauchy principal value of $\int_{-\infty}^{\infty} \frac{\sin x}{x^2+6x+10}dx$ (5%)