國立清華大學 102 學年度碩士班考試入學試題

系所班組別：核子工程與科學研究所 甲組(工程組)

考試科目（代碼）：近代物理(2703)

Useful constants: $c = 3.00\times10^{8}$ m/s, $k_B = 1.38\times10^{-23}$ J/K, $h = 6.63\times10^{-34}$ J·s,
$e = 1.60\times10^{-19}$ C, $m_e = 9.11\times10^{-31}$ Kg, $\mu_B = 9.274\times10^{-24}$ J/T,
Coulomb constant $k = 8.988\times10^{9}\ \mathrm{N\cdot m^2/C^2}$.

Part A. *Please write down the answers briefly for problems 1-5 (no need to write down the derivations):*

1. (6%) A certain particle has a lifetime of 1.0×10^{-8} s when measured at rest. How far does it go (on average) before decaying if its speed is 0.99 c when it is created?

2. (12%) A particle of unknown mass M decays into two particles of known mass $m_1 = 0.5\,\mathrm{GeV}/c^2$ and $m_2 = 1.0\,\mathrm{GeV}/c^2$, whose momenta are measured to be $p_1 = 2\,\mathrm{GeV}/c$ along the positive y axis and $p_2 = 1.5\,\mathrm{GeV}/c$ along the x axis. Find the unknown mass M (in units of GeV/c^2) and its speed (in units of c).

3. (12%) An X-ray diffractometer could not set the glancing angles below $\theta = 30^\circ$. A crystal analyzed by this X-ray diffractometer, with X-ray wavelength of 0.0438 nm, is found to have diffraction peaks at $\theta = 36.7^\circ$, 52.8°, and 84.5°. What are the orders of these peaks and what is the spacing of the crystal planes (in nm)?

4. (10%) An atom in an excited state 2.0 eV above the ground state remains in that excited state 2.0 μs before moving to the ground state. Find **(a)** the frequency and wavelength of the emitted photon, **(b)** its approximate uncertainty of the energy.

5. (10%) Rutherford shows a breakdown in the Rutherford scattering equation for 7.7 MeV α particles scattered at large angles from aluminum nucleus ($Z = 13$). Estimate the radius of the aluminum radius (in units of m) from these facts.

Part B. *Please just fill in those blank underlines (you do not need to write down the equations or the way you do calculations. We will not go through them):*

6. Please write down the probability, P, of finding a particle in a finite interval $a \leqq x \leqq b$ of a wave function $\Psi(x,t)$. P=_____ (3%)

7. If the initial wave function of a particle is given as $\Psi(x,0) = A\cdot\exp(-|x|/\xi)$, where A is the amplitude of the wave function and ξ is the characteristic length (a constant). Find the value of A in terms of ξ, such that the wave function is normalized.
A = _____ (4%)

8. For the non-relativistic free particle with mass m and wave number k, what is the relation between the frequency ω and wave number of wave function?
ω=_____(3%)

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9. Please write down a wave function of a plane wave for a free particle in terms of k, x, ω, t _____(3%)

10. Please write the Hamiltonian of a time independent Schrödinger equation. [H]= _____(3%)

11. Calculate the quantum of energy of the vibration of a nitrogen molecule if it behaves as a harmonic oscillator with force constant between hydrogen atoms being K=800 N/m and the reduced mass μ =5.8x10^{-27} kg. The quantum of energy $h\omega/2\pi$ =_____eV (4%)

12. When two copper wires are connected by twisting ends together, the electricity can pass through the twisted connection. However, as we all know that the surface of copper always consists of a thin layer (say 2 nm) of copper oxide which apparently is an insulator as a bulk material. But, still we can transport electricity from one wire to another wire that has only surface contacts through the twisted connection. Why is that? Because this is a _____phenomena . (3%)

13. Whenever different quantum states have the same energy, this energy level said to be _____.(3%)

14. In the quantum mechanics of three dimension, the orbital quantum number $l = 2$, what is the angle between the angular moment with $m_l = 0$ and $m_l = 1$? The angle θ =_____degrees.(3%)

15. If an electron has an orbit angular moment of 3.651x10^{-34} J·s what is the orbital quantum number for this state of the electron? l = _____.(3%)

16. In a central force system of atomic hydrogen, the energy E, and angular moment $\boldsymbol{L}$ of the system have sharp values. In this way, the $\boldsymbol{L}$ has three components, L_x, L_y, and L_z. Usually, only L_z has sharp value. Why is that? Because it should follow the_____.(3%)

17. In the de-excitation of atomic hydrogen by a photon emission, the $3p$ --> $2p$ transition is said to be forbidden, why is that? Because the transition must follow the _____. (3%)

18. What is the ratio between the "Bohr magneton" and "gyromagnetic ratio" for an atomic moment? The ratio is _____(3%)

19. In a Zeeman splitting for the ground state (n=1, l=0) and first excitation state (n=2, l=1) of a hydrogen atom immersed in a magnetic field B. An electron in one of the excited states decays to the ground state with the emission of a photon, giving rise to emission lines at frequencies of ω_0, $\omega_0+\omega_L$ and $\omega_0-\omega_L$. Now, if I applied the magnetic field in $2B$, what is the frequency of emission lines. (The spin is negligible here). The three frequencies are _____(3%)

20. What is the discovery of Stern-Gerlach experiment? They discovered _____.(3%)

21. Please write down the electronic configuration of a Fe atom in ground state in terms of s, p, d _____. (3%)