

國立清華大學命題紙

95 學年度 工程與系統科學 系(所) 乙、丙、戊 組碩士班入學考試

科目 工程數學 科目代碼 ³⁴⁰¹₃₅₀₂³⁶⁰¹ 共 1 頁第 1 頁 *請在【答案卷卡】內作答

1. $y(x) = e^{-x}$ is a solution of

$$(x-1)y'' - xy' + y(x) = 0.$$

Using the method of reduction of order, find another linear independent solution. (10%)

2. Using Laplace transform solve the boundary value problem

$$y'' - 2y' + y(x) = x, \quad y(0) = 0, \quad y'(1) = -2. \quad (11\%)$$

3. Find the general solution of the following differential equation and show the details.

$$xy'' - 2y' + xy(x) = 0, \quad 0 < x < \infty \quad (15\%)$$

4. Verify the divergence theorem by working out the theorem with the giving vector function \vec{V} and the volume V , where

$$\vec{V} = z^2 \vec{e}_z \text{ and } V: \text{the cone } r \leq 2z, 0 \leq \theta < 2\pi, 0 \leq z \leq 3$$

\vec{e}_z is the unit vector in the direction of z . (16%)

5. Use the method of diagonalization to obtain the general solution of the following equations:

$$x' + 2x + y = 0$$

$$y' + x + 2y + z = 0 \quad \text{where primes denote } d/dt. \quad (16\%)$$

$$z' + y + 2z = 0$$

6. Solve the diffusion equation

$$\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2} \quad \text{for } 0 < x < 1, t > 0$$

subject to the initial condition

$$u(x, 0) = 1 \quad \text{for } 0 < x < 1,$$

and the boundary conditions

$$u(0, t) = 0, \quad \left. \frac{\partial u}{\partial x} \right|_{x=1} = -h u(1, t) \quad \text{for } t > 0, \quad h > 0. \quad (16\%)$$

7. Compute

$$\int_{-\infty}^{\infty} \frac{x \cos x}{x^2 - 3x + 2} dx \quad (16\%)$$