國 立 清 華 大 學 命 題 紙

九十三學年度 工科 系（所）乙、丙、丁、戊 組碩士班入學考試

科目 工程數學 科號 3901、4002、4101、4201 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

1. (15%) (a) Find the particular solution of the ordinary differential equation

$$y''(x) + y(x) = \cos x \ , \qquad |x| < \infty \qquad (1)$$

(b) Find the solution of Eq.(1) with initial conditions $y(0) = 0$, and $y' = 1$.

2. (15%) If J_ν is a solution of the Bessel's equation

$$x^2 y''(x) + xy'(x) + (x^2 - \nu^2)y(x) = 0, |x| < \infty$$

Show that

(a) $J_\nu(\alpha t)$ satisfies the equation

$$\frac{d}{dt}\left[t\frac{d}{dt}J_\nu(\alpha t)\right] + (\alpha^2 t - \nu^2/t)J_\nu(\alpha t) = 0$$

(b) $\int_0^1 tJ_\nu(\alpha t)J_\nu(\beta t)dt = 0$, where α and β are two distinct roots of $J_\nu(x) = 0$, (i.e. $J_\nu(\alpha) = J_\nu(\beta) = 0$ and $\alpha \neq \beta$).

3. (10%) Prove that vectors $\vec{u}, \vec{v}, \vec{w}$, are linearly dependent if and only if $\vec{u} \cdot \vec{v} \times \vec{w} = 0$.

4 (10%) Determine the "?" integration limits.

$$\int_0^9 \int_{z/3}^{\sqrt{z}} \int_0^{y+z} f(x,y,z)dxdydz = \int_?^? \int_?^? \int_?^? f(x,y,z)dydzdx$$

5. (10%) Solve by Fourier cosine or sine transform

$$u'' - 16u = 50e^{-2x}, 0 < x < \infty \quad \text{with} \quad u'(0) = a, u(\infty)\ bounded$$

6. (10%) Solve the eigenvalues and eigenfunctions for

$y'' - 5y' + \lambda y = 0,\ (0 < x < \pi)$ as a Sturm-Liouville problem

with $y(0) = 0$ and $y(\pi) = 0$

7. (15%) Suppose that a solid right circular cylinder of radius a is of infinite extent on one side of the plane face z = 0, and that the temperature is maintained at zero along the lateral boundary, whereas the temperature distribution over the face z = 0 is prescribed as T(r,0) = f(r). Find the steady-state, axisymmetrical temperature distribution inside the cylinder.

[cylindrical coordinates (r,θ,z)

$$\nabla^2 T = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2}\ ]$$

8. (15%) Find the value of the integral

$$\int_0^{\infty} \frac{\sin \pi x}{x(1-x^2)}\,dx.$$