類組: <u>電機類</u> 科目: <u>電磁學 A(3007)</u>

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※請在答案卷內作答

1. (5%)

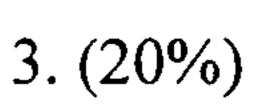
Find the electrostatic energy required to assemble a sphere of charge of radius b and the following volume charge density,

$$\rho = \begin{cases} \frac{\rho_0 R}{b}, & 0 \le R \le b \\ 0, & R > b \end{cases}$$
, where ρ_0 is a constant.

2. (15%)

A point charge Q is at a distance d from the center of a grounded conducting sphere of radius a (a < d). By using the method of images,

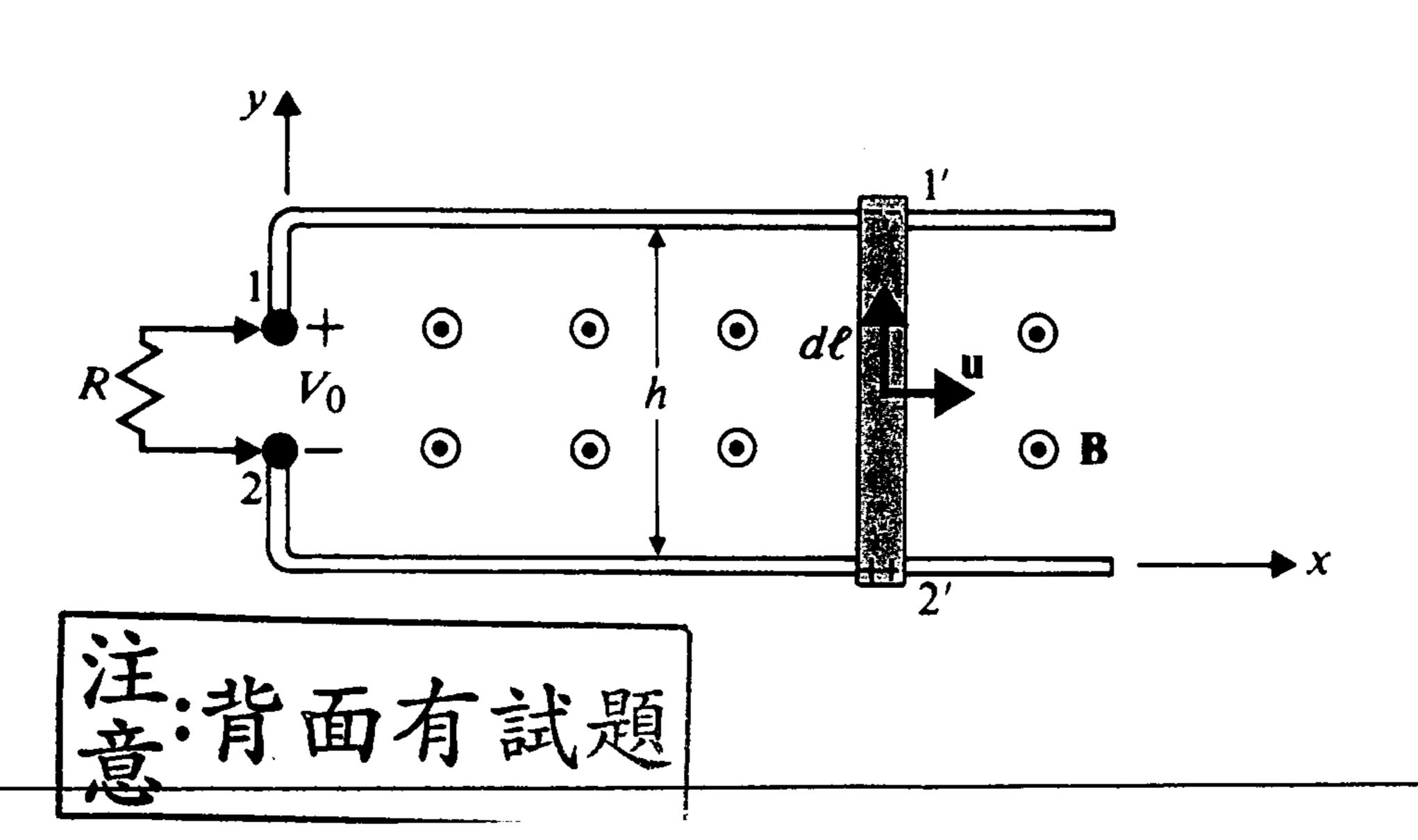
- (a) Find the location and quantity of the image charge Qi. (5%)
- (b) Find the charge distribution induced on the sphere surface. (5%)
- (c) Calculate the total charge induced on the sphere. (5%)



A metal bar slides over a pair of conducting rails in a uniform magnetic field

 $\vec{B} = B_0 \cdot \hat{a}_z$, with a constant velocity \vec{u} , as shown in following figure.

- (a) Determine the open-circuit voltage V₀ that appears across terminals 1 and 2. (5%)
- (b) Assuming that a resistance R is connected between the terminals, find the electric power dissipated in R. (5%)
- (c) Show that this electric power is equal to the mechanical power required to move this sliding bar with a velocity \vec{u} . (Neglect the electric resistance of the metal bar and conducting rails, and all friction). (10%)



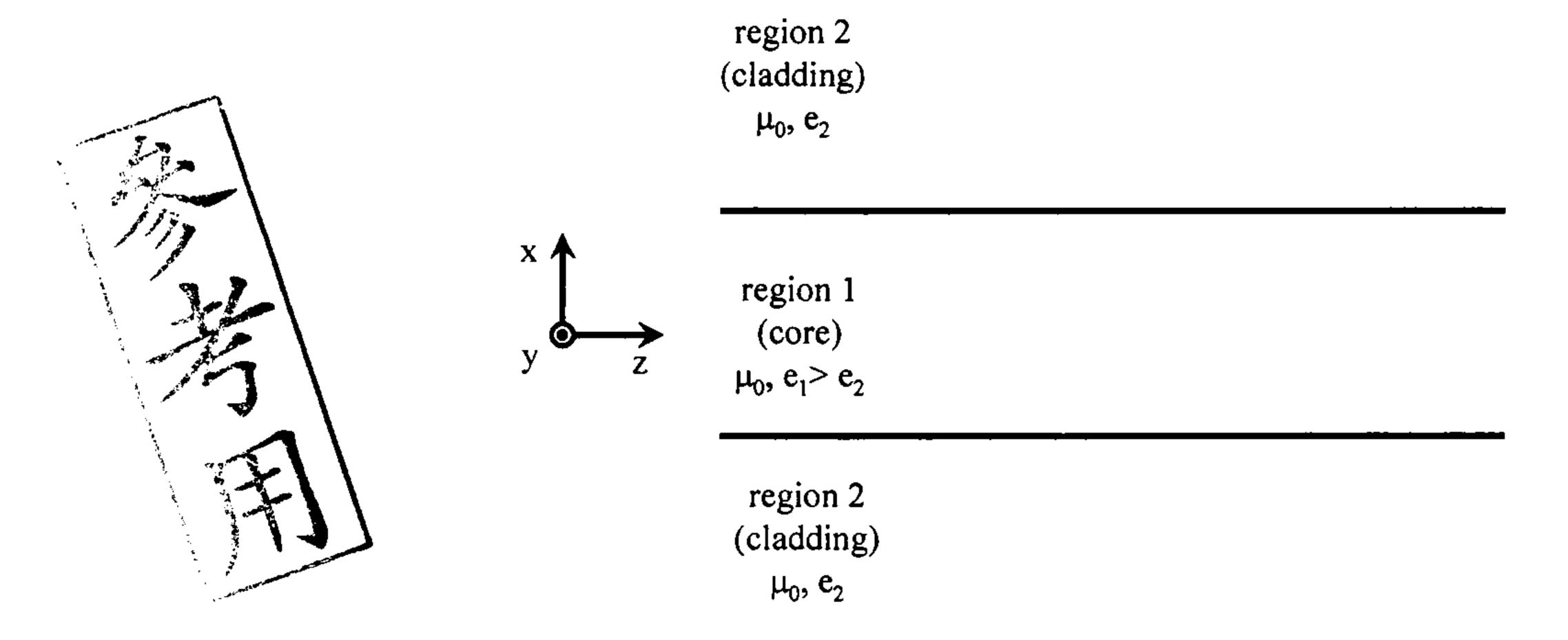
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※請在答案卷內作答

4. (15%)

Consider a symmetric dielectric slab waveguide where a dielectric slab (core) is surrounded by another dielectric material (cladding) with a lower permittivity as shown below. Assume the wave propagates in z direction and there is no variation of fields in the y direction.



- (a) The field components of modes can be solved from the wave equation with boundary conditions. For a mode guided within the slab, what field variations along x should be considered in the slab and in the cladding? Please also discuss the meaning. (4%)
- (b) Please draw the electric field distribution of the fundamental mode over the transverse plane directly on the figure above and discuss the associated cut-off condition. (3%)
- (c) Assume the core thickness is d and free-space wavelength is λ_0 . If one increases the ratio of d/λ_0 , is this waveguide more likely to have more modes or fewer modes? Please explain. (4%)
- (d) When operated from near cut-off to far from cut-off, will a mode be more or less tightly bound to the slab? Please explain. (4%)

5. (15%) Transmission Lines

- (a) Determine the magnitude of the reflection coefficient, $|\Gamma|$, of a lossless transmission line connected to a purely reactive load. (5%)
- (b) Find the two shortest lengths of a shorted 50 Ω lossless transmission line such that their input impedances at 2.25 GHz are identical to that of a capacitor with capacitance $C_{eq} = 4$ pF. The wave velocity on the line is 0.75c, where $c = 3 \times 10^8$ m/s. You may express your answers in terms of π and arc-tangents. (10%)

注:背面有試題

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※請在答案卷內作答

6. (15%)

Consider a z-oriented hollow rectangular metallic waveguide with uniform cross section of width a (along the x direction) and height b (along the y direction), and it is given that a > b.

- (a) Write down the cutoff frequency for the TM_{mn} mode and TE_{mn} mode. (3%, 3%)
- (b) Plot qualitatively typical field lines for TE_{10} mode within the waveguide on the x-y, z-y and z-x planes which pass through the center of the waveguide, respectively. Please use solid (dashed) lines for the electric (magnetic) field in your plots. (3%, 3%, 3%)

7. (15%)

A source-free homogeneous vector Helmholtz's equation of E-field can be written as:

 $\nabla^2 E + k_c^2 E = 0$, where $k_c = \omega \sqrt{\mu(\epsilon - j\frac{\sigma}{\omega})}$. Please define (a) propagation constant, (b)

2%, 2%)

