國立清華大學 104 學年度碩士班考試入學試題

系所班組別：聯合招生 (0598)

考試科目（代碼）：電磁學 (9803)

　　*請在【答案卷】作答

請注意：電磁常數給於題目卷最後一頁

1. Please answer the questions in MKSA system. (10%)

(a) Let $\vec{E}$, $\vec{D}$, $\vec{H}$, $\vec{B}$, ρ, $\vec{J}$ denote the electric field intensity vector, electric flux density vector, magnetic field intensity vector, magnetic flux density vector, volume charge density (free), and current density vector, respectively. Complete the Maxwell's equations in matter

$\nabla \cdot \vec{D}$ =________________

$\nabla \times \vec{E}$ =________________

$\nabla \cdot \vec{B}$ =________________

$\nabla \times \vec{H}$ =________________

(b) Express $\vec{E}$ and $\vec{B}$ in terms of the electric scalar potential V and the magnetic vector potential $\vec{A}$

$\vec{E}$ =____________________

$\vec{B}$ =____________________

(c) A partice of chare q moving with velocity $\vec{v}$ in the presence of $\vec{E}$ and $\vec{B}$ is subjected to a Lorentz force:

$\vec{F}$ =____________________

(d) Poynting vector $\vec{S}$ represents the directional energy flux density of an electromagnetic field. Give the definition of $\vec{S}$

$\vec{S}$ =____________________

Poynting theorem, a statement of conservation of energy for electromagnetic field, can be summarized in a differential form:

$\frac{\partial u}{\partial t}$ =____________________

where u =____________________ is the electromagnetic energy density.

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2. A coaxial cable with perfect conductors is filled with an inhomogeneous imperfect dielectric of permittivity $\varepsilon = \varepsilon_0(r+a)/a$ and conductivity $\sigma_d = \sigma_0 a/(r+a)$, where a and b are the radius of the inner cable conductor and the inner radius of the outer conductor. ε_0 is vacuum permittivity and σ_0 is constant. The DC voltage between the cable conductors is V. Calculate

(a) the conductance per unit length of the cable, G' (5%)

(b) the electric flux density vector $D(r)$, $a \le r \le b$, in the cable (5%)

(c) the density of volume free charges $\rho(r)$ in the dielectric (5%)

Formula: in cylindrical coordinates, $\operatorname{div}\vec{v} = \frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial}{\partial \phi} v_\phi + \frac{\partial}{\partial z} v_z$

3. The dielectric of a spherical capacitor consists of two concentric layers. The relative permittivity of the inner layer is $\varepsilon_{r1} = 4$ and its dielectric critical strength is $E_{cr1} = 60$ MV/m. For the outer layer, $\varepsilon_{r2} = 8$ and $E_{cr2} = 30$ MV/m. Electrode radii are $a = 4$ cm and $c = 8$ cm. The radius of the boundary surface between the two layers is $b = 6$ cm.

(a) Which layer breaks down first when we increase charge on the electrodes? (5%)

(b) Calculate the breakdown voltage V_{cr} of the capacitor. (5%)

4. (a) Describe the Biot-Savart law and give the formula. (5%)

(b) Consider a circular loop of radius a carrying a steady current of intensity I in air. The loop is located on the xy-plane and has the z-axis as its axis. Calculate the magnetic flux density vector $\vec{B}$ along the z-axis. (10%)

5. Using Ampere's law, calculate the magnetic field intensity vector $H(r)$ inside and outside an infinitely long cylinder of radius a which is uniformly charge by a charge density ρ and rotates about its axis with an angular velocity ω. (10%)

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6. A conducting rod rotates in a uniform magnetic field with an angular velocity ω about an axis which is perpendicular to the rod and splits the rod onto two unequal parts of lengths ℓ_1 and ℓ_2. The magnetic field varies slowly with time and its flux intensity is $B(t) = B_0 \sin(\omega_0 t)$, $\omega \gg \omega_0$, parallel to the axis. Find the instantaneous induced electromotive force measured between the two ends of the rod. (10%)

7. Consider a linear conductor of permittivity ε and permeability μ. An electromagnetic wave of angular frequency ω propagates along z-direction in the conductor. Assume $\vec{J} = \sigma\vec{E}$ and $\rho = 0$. We search for the plane-wave solution by assuming $\tilde{E} = \tilde{E}_0 \exp\, i(\tilde{k}z - \omega t)$, $\tilde{H} = \tilde{H}_0 \exp\, i(\tilde{k}z - \omega t)$ in the complex space.

(a) Show that when solving the problem, the permittivity is effectively equal to $\tilde{\varepsilon} = \varepsilon + i\frac{\sigma}{\omega}$. (5%)

(b) The wave number $\tilde{k}$ is a complex number and equal to $k' + ik''$ where $k' = \omega\sqrt{\frac{\varepsilon\mu}{2}}\sqrt{u+1}$, $k'' = \omega\sqrt{\frac{\varepsilon\mu}{2}}\sqrt{u-1}$. Find u=? (5%)

(c) The electric field and the magnetic field are no longer in phase. Find the phase difference ϕ=? in term of u. (5%)

8. (a) It is shown that for the propagation of an electromagnetic wave of frequency f, a plasma medium has an equivalent permittivity $\varepsilon_p = \varepsilon_0(1 - \frac{f_p^2}{f^2})$ where $f_p = \frac{1}{2\pi}\sqrt{\frac{nq^2}{\varepsilon_0 m}}$ is the plasma frequency. Explain why a plasma medium behaves as a high-pass filter for electromagnetic waves. (5%)

(b) The cutoff frequency for TE_{mn} and TM_{mn} modes in a rectangular waveguide of dimension $a \times b$ is $f_{mn} = \frac{v_p}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$ where $v_p = 1/\sqrt{\varepsilon\mu}$ is the phase velocity of a TEM wave. Consider a waveguide of $a = 20\text{cm}$ and $b = 10\text{cm}$, filled with a nonmagnetic dielectric with relative permittivity $\varepsilon_r = 9$. What wave modes can propagate along this waveguide at a frequency $f = 0.8$ GHz? (10%)

Constants:

Vacuum permittivity $\varepsilon_0 = 8.8542 \times 10^{-12}$ F/m

Vacuum permeability $\mu_0 = 4\pi \times 10^{-7}$ H/m

Light speed in vacuum $c_0 = 3 \times 10^{8}$ m/s