國　立　清　華　大　學　命　題　紙

99 學年度生命科學院丙組碩士班入學考試

科目＿＿微積分＿＿科目代碼＿0401＿共＿1＿頁第＿1＿頁 *請在【答案卷】內作答

(1) Evaluate the following integrals

10% (i) $\int_0^\infty e^{-\sqrt{x}}dx$

15%(ii) $\int \frac{x}{x^3-1}dx$

10%(iii) $\iint_D \sin\sqrt{x^2+y^2}\,dxdy$ where $D=\{(x,y):\pi^2\le x^2+y^2\le 4\pi^2\}$

10%(iv) $\int_{-\infty}^{\infty} x^2e^{-x^2}\,dx$

10% (2) Plot the graph of $y=\frac{xe^x}{e^x-1}$, $x\neq 0$

10% (3) Find the following limit

$$\lim_{x\to\infty}\left(\sin\frac{1}{x}+\cos\frac{1}{x}\right)^x$$

10% (4) Assume $\lim_{x\to\infty}\left(\frac{x^2+1}{x+1}-ax-b\right)=0$. Find the constants a and b.

15% (5) Let $f(x)$ be a positive strictly decreasing function. Show that if $\sum_{k=1}^{\infty} f(k)$ converges then $R_n=\sum_{k=n+1}^{\infty} f(k)$ satisfies

$$\int_{n+1}^{\infty} f(x)dx < R_n < f(n+1)+\int_{n+1}^{\infty} f(x)dx.$$

Use above to estimate the sum of $\sum_{n=1}^{\infty}\frac{1}{n^3}$ with accuracy 0.01.

10%(6) Let $z=e^y\varphi\left(ye^{\frac{x^2}{2y^2}}\right)$ where $\varphi=\varphi(\xi)$ is a differentiable function.

Show that

$$\left(x^2-y^2\right)\frac{\partial z}{\partial x}+xy\frac{\partial z}{\partial y}=xyz.$$