國　立　清　華　大　學　命　題　紙

98 學年度＿＿＿＿＿生命科學院＿＿＿＿＿系（所）＿＿丙＿＿組碩士班入學考試

科目＿＿＿微積分＿＿＿科目代碼＿0401＿共＿1＿頁第＿1＿頁 *請在【答案卷】內作答

I. Evaluate the following integrals

10%(1) $\int_2^e \frac{1}{x^3 - x^2} dx$

10%(2) $\iint_D (3x + 4y^2)\, dxdy$ where $D = \{(x, y) : y \geq 0, 1 \leq x^2 + y^2 \leq 9\}$

10%(3) $\oint_C (x + y)\, dx - (x - y)\, dy$ where $C = \left\{(x, y) : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\right\}$

10%(4) $\int_1^e \frac{\ln x}{x^2}\, dx$

II. Evaluate the following limits

10%(5) $\lim_{x \to 0} \frac{1 - \sqrt{\cos x}}{1 - \cos(\sqrt{x})}$

10%(6) $\lim_{n \to \infty} \frac{\sqrt[n]{n!}}{n}$

III.

10%(7) Let $x \in \mathbb{R}$, $a \in \mathbb{R}$, $f(x) = xe^{-x^2}$. Discuss how many solutions the equation $f(x) = a$ has as the parameter a varies.

10%(8) Suppose $f : \mathbb{R} \to \mathbb{R}$ is continuous. Define $G(x) = \int_0^x (x - t) f(t) dt$. Prove that $G''(x) = f(x)$ for all x.

10%(9) Let $f(x, y) = e^{xy} + x^2 + 2xy$, $\phi(t) = f\left(2t, e^t\right)$. Compute $\phi''(0)$.

10%(10) Let $a > 0$, $\alpha, \beta, \gamma > 1$. Find the extreme values of $f(x, y, z) = x^\alpha y^\beta z^\gamma$ subject to the constrain $x + y + z = a$.