國　立　清　華　大　學　命　題　紙

96 學年度＿＿＿＿＿生命科學院＿＿＿＿＿系（所）＿＿丙＿＿組碩士班入學考試

科目＿＿微積分＿＿科目代碼＿0401＿共 3 頁第 1 頁 *請在【答案卷】內作答

(1) Evaluate the following limits

5% (a) $\lim_{x\to 0^+} x^{\sin x}$

5% (b) $\lim_{x\to\infty}\left(\sqrt{x^2+2x}-x\right)$

10% (2) Evaluate the integral $\int_a^\infty \frac{dx}{\left(r^2+x^2\right)^{3/2}}$, $r>0$, $a>0$.

10% (3) Prove by induction $\int_0^\infty x^n e^{-x}dx = n!$ where n is a positive integer.

10% (4) Fit a line $y=mx+b$ to a given set of numerical data $(x_1,y_1),\cdots,(x_n,y_n)$ by minimizing the sum of squares (See Figure 1)

$$f(m,b)=(mx_1+b-y_1)^2+\cdots+(mx_n+b-y_n)^2.$$

Show that the minimum is achieved at

$$m^*=\frac{(\sum x_k)(\sum y_k)-n\sum x_k y_k}{(\sum x_k)^2-n\sum x_k^2},$$

$$b^*=\frac{1}{n}\left(\sum y_k - m^*\sum x_k\right)$$

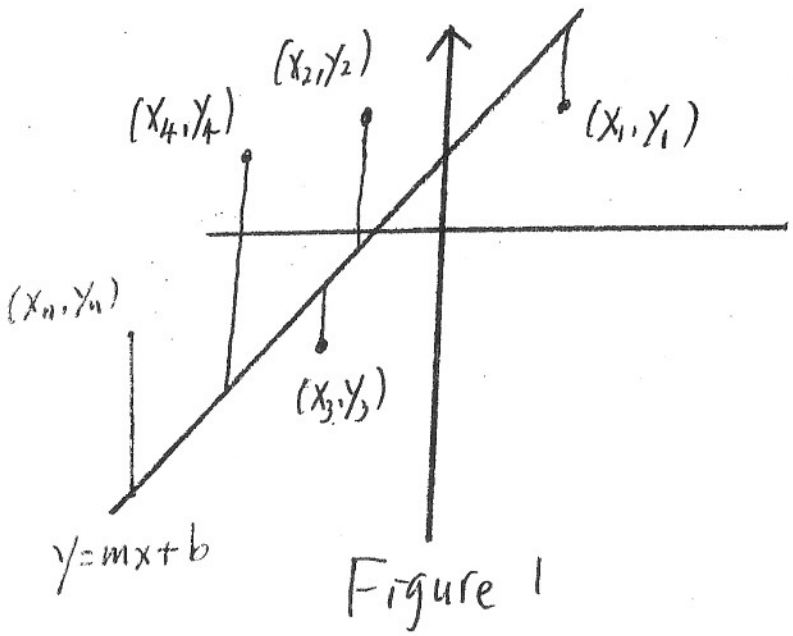

Figure 1

(5) 8% (i) Prove $c(x,t) = \dfrac{1}{\sqrt{4\pi Dt}} \exp\left(-\dfrac{x^2}{4Dt}\right)$ satisfies the diffusion equation

$$\frac{\partial c}{\partial t}(x,t) = D\frac{\partial^2 c(x,t)}{\partial x^2}$$

7% (ii) Show that $\int_{-\infty}^{\infty} c(x,t)dx = 1$, for $t > 0$ by using the fact $\int_{-\infty}^{\infty} e^{-u^2} du = \sqrt{\pi}$.

10% (6) Evaluate the double integral

$$\int_0^1 \int_y^1 \frac{1}{\left(1+x^2+y^2\right)^{3/2}} dxdy$$

(Hint: use polar coordinate).

(7) Assume the concentration of a drug in the blood is $f(t)=ate^{-bt}$,

$a>0$, $b>0$ where t is the time after taking the drug.

8% (i) Assume the concentration of drug reaches its maximum $3\times10^{-3}\,gram/liter$ at $t=2hrs$. What is the concentration of the drug at $t=6hrs$.

7% (ii) Sketch the graph of $x=f(t)$ for $t\geq 0$

10% (8) Evaluate the line integral

$$\int_C \left(2y-e^x\cos y\right)dx+\left(e^x\sin y+2x\right)dy$$

where C is the curve of the upper semicircle $x^2+y^2=ax$ from point $A(a,0)$ to origin $O(0,0)$ (See Figure 2)

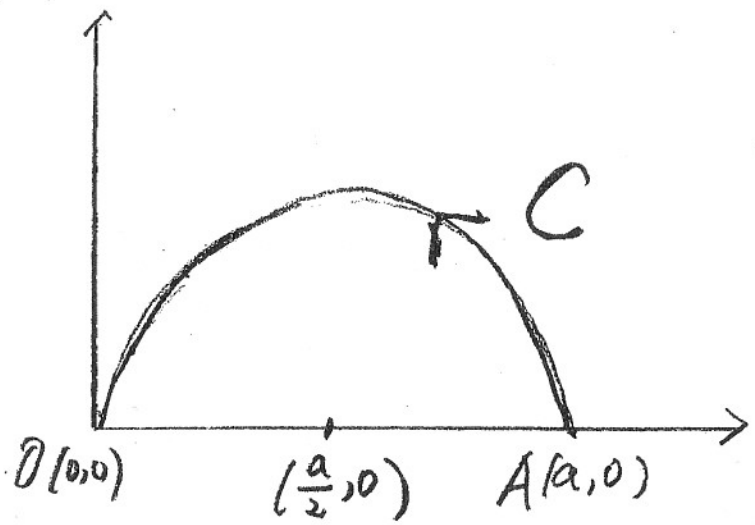

Figure 2

10% (9) Let $f:\mathbb{R}^3\to\mathbb{R}$ be a differentiable function. Explain the geometric meaning and the physical meaning of the gradient vector $\nabla f(\vec{x}_0)$.