(1) Compute the following integrals

5% (i) $\int x^2 \tan^{-1} x dx$

5% (ii) $\int_0^1 x\,\ell n\, x\, dx$

5% (iii) $\int \frac{x+3}{x^2-3x+2} dx$

(2) Evaluate the following limits

5% (i) $\lim_{x\to 0}\left(e^x+3x\right)^{\frac{1}{x}}$

5% (ii) $\lim_{x\to 0}\left(\frac{1}{\ell n(1+x)}-\frac{1}{x}\right)$

(3) Evaluate the integral

10% $\int_{-2}^{4}\int_{-2}^{\frac{1}{2}(y-2)}\left[y+\sin\left(3x^2+12x-7\right)\right]dxdy$ by changing order of integration

(4) 15% Find maximum and minimum value of the function

$f(x,y,z)=-2x+2y+3z$ subject to the constrains

$-x+2y+z=0,\qquad 4\left(y+1\right)^2+z^2=16$

(5)(i) Let $n\geq 3$ be positive integer and

$$f(x_1,\cdots,x_n)=(x_1^2+\cdots+x_n^2)^{\frac{2-n}{2}}$$

7% Show that f satisfies Laplace equation

$$\frac{\partial^2 f}{\partial x_1^2}+\cdots+\frac{\partial^2 f}{\partial x_n^2}=0 \quad \text{for} \quad (x_1,\cdots,x_n)\neq(0\cdots 0).$$

(ii) Let $u = u(x, y)$ have continuous second partial derivatives.

$x = r\cos\theta, y = r\sin\theta$.

8% Show that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} + \frac{1}{r}\frac{\partial u}{\partial r}.$$

(6) 10% Let Ω be the region bounded by paraboloid $z = x^2 + y^2 + 4$ and the plane $z = 20$. Let $n > 0$ be positive integer. Evaluate the integral

$$\iiint_{\Omega} \left(x^2 + y^2\right)^n dV.$$

(7) Let $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, -\infty < x < \infty.$ Show that

7% (i) $\int_{-\infty}^{\infty} f(x)dx = 1$

8% (ii) $\mu = \int_{-\infty}^{\infty} xf(x)dx.$

(8)

5%(i) Find $F'(x)$ if $F(x) = \int_{x^2}^{\sin x} \left(t - \sin t^2\right) dt$

5%(ii) Find $F'(1)$ and $F''(1)$ if

$$F(x) = \int_0^{x^2} \left[t \int_1^t f(u)du \right] dt$$