Planck's constant $h = 6.626 \times 10^{-34}\, J\ \text{sec}$ (J = Joule)

Coulomb's low constant $\dfrac{1}{4\pi\varepsilon_0} = 8.998 \times 10^9\ Nt - m^2/Coul^2$

Bohr magneton $\mu_B = \dfrac{e\hbar}{2m_e} = 9.27 \times 10^{-24}\ J/T$

Bohr radius $a_0 = \dfrac{4\pi\varepsilon_0 \hbar^2}{m_e e^2} = 5.29 \times 10^{-11}\, m$

Bohr energy $E_1 = \dfrac{-m_e e^4}{(4\pi\varepsilon_0)^2 (2\hbar)^2} = 13.6eV$

Boltzmann's constant $k_B = 1.38 \times 10^{-23}\ J/K$

$1eV = 1.602 \times 10^{-19}\, J$ $W = J/\text{sec}$

$m_e c^2 = 0.511 MeV$ $c = 2.997 \times 10^8\ m/\text{sec}$

Useful definite integrals

$$\int_0^a \chi \sin^2 \frac{n\pi\chi}{a} d\chi = \left(\frac{a}{2}\right)^2 \qquad \int_0^a \sin\frac{\pi\chi}{2a} \sin\frac{\pi\chi}{a} d\chi = \frac{4a}{3\pi}$$

$$\int_0^a \chi^2 \sin^2 \frac{n\pi\chi}{a} d\chi = \frac{a^3}{6} - \frac{a^2}{4n^2\pi^2}$$

Trigonometric relationships

$$\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$

$$\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

Lorentz transformation

$$x = r(x' + \upsilon' t) \qquad x' = r(x - \upsilon t)$$

$$y = y' \qquad y' = y \qquad r = \frac{1}{\sqrt{1 - (\upsilon^2/c^2)}}$$

$$z = z' \qquad z' = z$$

$$t = r\left(t' + \frac{\upsilon}{c^2} x'\right) \qquad t' = r\left(t - \frac{\upsilon}{c^2} x\right)$$

υ = speed

Part I (50%)

There are 10 blanks A to J in this part. Each gets 5 points (5%). Please list the answers of each blank on the first page of your answer sheets.

1. Two events occur at the same point x_0' at time t_1' and t_2' in S′ which moves with speed υ relative to S. The spatial separation of these events measured in S is ____(A)____.

2. At NSRRC, electrons are accelerated to energies of 1.5 GeV.

   $\frac{c-\upsilon}{\upsilon}=$ ____(B)____.

   (v is the speed of electrons, c is the velocity of light)

3. A monochromatic point source radiates light of 500 nm wavelength at a rate of 20 mW uniformly in all directions. The number of photons enter the 2 mm diameter pupil of an observer stationed at 2 km from the source is ____(C)____.

4. The magnetic moment due to orbital motion of the electron in the first Bohr orbit is ____(D)____.

5. A monoenergetic beam of electrons is incident on a single slit of width 0.5 nm. A diffraction pattern is formed on a screen 20 cm from the slit. If the distance between successive minima of diffraction pattern is 2.1 cm. The energy of the incident electron is ____(E)____.

6. Robert Hofstader won the 1961 Nobel Prize in physics for the pioneering work in scattering 20 GeV electrons from nuclei.

The de Brogile wavelength of a 20 GeV electron is ____(F)____.

The size of a nucleus with A = 216 is approximately ____(G)____.

7. An electron in the Coulomb field of a proton is in a state described by the wavefunction

$$\frac{1}{6}\left[4\Psi_{100}(\vec{r})+3\Psi_{211}(\vec{r})-\Psi_{210}(\vec{r})+\sqrt{10}\Psi_{2,1,-1}(\vec{r})\right]$$

[$\Psi_{n,l,m}(\vec{r})$ are the eigenfunctions of $H, \vec{L}_2, L_z$ with eigenvalues $-\frac{13.6eV}{n^2}$, $l(l+1)\hbar^2$, $m\hbar$, respectively]

The expectation value of energy is ____(H)____.

The expectation value of $\vec{L}_2$ is ____(I)____.

The expectation value of L_z is ____(J)____.

Part II (50%)

There are four problems in this part. Please write down the answers start from the second page of your answer sheets.

8. Consider a particle moving freely in a one dimensional box of length L, i.e.,

$$U(x) = \begin{cases} 0 & 0 \le x \le L \\ \infty & x < o, x > L \end{cases} \qquad (15\%)$$

(1) Show the eigenfunctions are given by $\Phi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$ with eigenvalues $E_n = \frac{\hbar^2 \pi^2 n^2}{2mL^2}$ n = 1, 2, 3, . ..

(2) For the ground state, find $\Delta x, \Delta p_x$

(3) Given $\Psi(x,0) = \frac{1}{\sqrt{2}}\Phi_1(x) + \frac{1}{\sqrt{2}}\Phi_2(x)$

Find $\Psi(x,t)$ and $\langle x \rangle_t$

9. Consider the normal Zeeman effect applied to the 3d to 2p transition. (10%)

(1) Sketch an energy level diagram that shows the splitting of the 3d and 2p levels in a magnetic field. Indicate all possible transitions from each m state of the 3d level to each m state of the 2p level.

(2) Which transitions satisfy the $\Delta m = \pm 1, 0$ selection rules? [s l = 0; p l = 1; d l = 2, .]

(3) Show that there are three different transition energies emitted.

10. Estimate the temperature at which $\frac{1}{1000}$ of the monoatomic hydrogen will be in the first excited state. (5%)

11. Give a concise explanation of the following techniques and methods (20%)

(1) X-ray diffraction and molecular structure.

(2) Nuclear magnetic resonance, nuclear magnetic imaging.

(3) Scanning tunneling microscopy.

(4) Optical tweezes.

Example for Question 11

Question: Fusion

Answer: The nuclear process whereby two light nuclei can overcome the mutual electric (Coulomb) repulsion to fuse together. This is accompanied by the released of a large amount of energy and is the source of energy in the Sun and other stars. It is hoped that fusion will one day be harnessed as an energy source on Earth.