

國 立 清 華 大 學 命 題 紙

九十三學年度 生科院丙組、生科院（結構生物學程）乙組 碩士班入學考試

科目 近代物理 科號 1002、1202 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

每格 5 分，共 100 分

答題時請列出計算過程與答案，數值僅需正確至三位有效數字

Physical Constant

$$e = 1.60218 \times 10^{-19} \text{ C}$$

$$\alpha = 1/137.036$$

$$k_B = 1.381 \times 10^{-23} \text{ J K}$$

$$a_0 = 5.292 \times 10^{-11} \text{ m}$$

$$m_e = 9.10939 \times 10^{-31} \text{ kg}$$

$$c = 2.9979 \times 10^8 \text{ m/s}$$

$$1/4\pi\epsilon_0 = 8.988 \times 10^9 \text{ N-m}^2/\text{C}^2$$

$$h = 6.626 \times 10^{-34} \text{ J s}$$

$$\ln 2 = 0.693$$

1. The de Broglie wavelength of a 200 keV photon is (1) . If an electron has the same wavelength, its energy (in eV) is (2) .
2. The 200 keV photon undergoes a Compton scattering and was detected at the scattering angle of 60° , the wavelength is changed by (3) . (Please use +/- to indicate increase/decrease of wavelength.)
3. Electrons of energy 1.5 GeV ($1.5 \times 10^9 \text{ eV}$) are moving at nearly the speed of light. Its speed is lower than c by a ratio of (4) .
4. Two particles are moving in the same direction of speed $0.9c$ and $0.6c$. The relative speed is (5) .
5. A particle moves in a one-dimensional infinite square potential well between $x=0$ and $x=a$. The energy eigenvalue for the ground state is (6) . The probability of this particle at ground state appear in the region of $0 \leq x \leq a/4$ is (7) .
6. Given a quantum system, let ϕ_g and ϕ_e be the normalized wave function for the lowest two states with eigenenergies E_g and E_e . Suppose a particle's wave function is described by $\phi = 1/\sqrt{5} \phi_g + 2/\sqrt{5} \phi_e$, the energy expectation value is (8) . The probability of finding the particle at ground state is (9) .

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7. A particle of mass m is under a potential of the form $V(x) = -\alpha \delta(x)$, where α is a constant. The bound state wave function is (10) and the energy eigen value is (11).
8. $A \exp(-\alpha x^2)$ is the ground state eigenfunction of a simple harmonic oscillator with the potential $V = \frac{1}{2} m \omega^2 x^2$, α is (12), the ground state energy is (13).
9. The ground state energy of a hydrogen atom is -13.6 eV. Neglecting the screening effect, the ionization energy for element K ($Z=19$) is (14) eV.
10. According to selection rules, E1 transition can happen from an electron in $4d_{3/2}$ state to a lower energy final states at (15). (Please list all possibilities.)
11. The three components of the angular momentum operator are defined as: $L_x \equiv y p_z - z p_y$, $L_y \equiv z p_x - x p_z$ and $L_z \equiv x p_y - y p_x$. The commutator of any two operators A and B is defined as $[A, B] \equiv AB - BA$. $[L_x, y] =$ (16), $[L_x, L_y] =$ (17).
12. Suppose protons do decay into neutrons, and the half-life is 10^{37} y. The amount of water (with 10 protons per H_2O molecule) required for measuring one decay per day is (18). In order to obtain a statistical uncertainty of 10% or better, the experiment should last for at least (19), under the assumption that the background contributes 2 counts per day in the detecting system.
13. A hydrogen atom at 1s state of wave function

$$\Psi_{100} = \frac{1}{\sqrt{\pi}} \left(\frac{1}{a_0} \right)^{3/2} e^{-r/a_0}$$

is under a small external electric field $E_0 z$ (along the z -direction). From perturbation theory the 1s energy is changed by (20)