

國立清華大學命題紙

九十二學年度 生命科學院 系(所) 丙 組碩士班研究生招生考試

科目 微積分 科號 1001 共 1 頁第 1 頁 *請在試卷【答案卷】內作答

1. Find the limits for the following:(20%)

$$\lim_{x \rightarrow 1} \frac{x^2 - 3x + 2}{x^2 - 5x + 4}$$

$$\lim_{x \rightarrow 2} \frac{x - \sqrt{3x - 2}}{\sqrt{x + 2} - 2}$$

$$\lim_{x \rightarrow \infty} (2x + 1 - \sqrt{4x^2 + 2x + 1})$$

$$\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3}$$

$$\lim_{\theta \rightarrow 0} \frac{1 - \cos 3\theta}{\theta^2}$$

2. Differentiate the following:(20%)

$$\cosh(\sinh x)$$

$$\sqrt{x \sin x \sqrt{1 - e^x}}$$

$$\frac{1 + x^2}{x\sqrt{1 - x^2}}$$

$$\frac{\sqrt{x^2 + x - 1}}{x} + \frac{1}{2} \arcsin \frac{x - 2}{\sqrt{5x}}$$

$$\frac{(x + 1)^3 \sqrt[4]{x - 2}}{\sqrt[5]{(x - 3)^2}}$$

3. Find the following integrals:(40%)

$$\int x(x^2 - 2)^4 dx$$

$$\int \frac{\tan x}{\cos^2 x} dx$$

$$\int 4x\sqrt{x^2 + 3} dx$$

$$\int \frac{x^2 + x + 1}{x^2 + 1} dx$$

$$\int \frac{x^2}{\sqrt{1 + x^3}} dx$$

$$\int \frac{4x^2 + 3}{(2x + 1)^2} dx$$

$$\int \frac{e^x}{\sqrt{1 + e^x}} dx$$

$$\int \frac{dx}{\sin x}$$

$$\int \frac{\sin^2 x}{1 + \sin^2 x} dx$$

$$\int \frac{dx}{e^x - e^{-x}}$$

4. Find the following integrals:(20%)

$$\int_0^{\pi/2} \sqrt{x \sin x} dx$$

$$\int_{-2}^{-1} \frac{dx}{(11 + 5x)^3}$$

$$\int_0^{a/2} \frac{adx}{(x - a)(x - 2a)}$$

$$\int_1^{\sqrt{3}} \frac{\sqrt{1 + x^2}}{x^2} dx$$

$$\int_0^2 \frac{dx}{x^2 - 4x + 3}$$