1. (18%) Compare Boltzmann, Bose and Fermi distributions in terms of their (a) basic characteristics (b) distribution functions and (c) examples.

2. (22%) An assembly of N particles of spin 1/2 are lined up on a straight line. Only nearest neighbors interact. When the spins of the neighbors are both up or down, their interaction is J. When one is up and the other is down, the interaction is -J. Prove that the partition function of the assembly is (k is the Boltzmann constant)

$$Z = 2^N \left[\cosh\left(\frac{J}{kT}\right) \right]^{N-1}$$

(Hint: $\sum_{k=0}^{N-1} \frac{(N-1)!}{[(N-1)-k]! k!} \exp[-kx] = (1 + \exp(-x))^{N-1}$)

3. (20%) Use the variational principle to estimate the ground-state energy of a particle in the potential

$$V = \infty \quad x < 0$$
$$V = cx \quad x > 0$$

Take xe^{-ax} as the trial function.

4. (40%) Write True (T) or False (F) for the following statements.

1. Diamagnetism is negative magnetic susceptibility.

2. Nuclear magnetic dipole moments are three orders of magnitude smaller than the electronic magnetic dipole moments.

3. Liquid helium is called He I when it is below the λ point (2.18 K) and He II when above.

4. The triplet state electrons act as if they repel each other, while the singlet state electrons acts the other way.

5. The *exchange force* present in electrons is a classical property.

6. For a good wavefunction, $\psi(x)$ and $\frac{d\psi(x)}{dx}$ should be continuous, but $\frac{d^2\psi(x)}{dx^2}$ is not required to be continuous.

7. While the zero-point energy of a harmonic oscillator is not zero, the zero-point energy of a particle-in-a-box is zero.

8. The energy of any quantum system is quantized.

9. It is possible to have a finite number of energy levels in a quantum system.

10. Bose condensation has been observed experimentally.