

八十七學年度 生命科學 系(所) 分生組丙 組碩士班研究生入學考試

微積分 科號 0901 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

1. Find the limits of the following: (20%)

a. $\lim_{x \rightarrow +\infty} \frac{\log x}{x}$

b. $\lim_{x \rightarrow +\infty} \frac{x^n}{2^x}$

c. $\lim_{x \rightarrow 0} x \log x$

d. $\lim_{x \rightarrow 0} \left(\frac{1}{x^2} - \frac{1}{\tan^2 x} \right)$

e. $\lim_{x \rightarrow +\infty} (1+x)^{\frac{1}{x}}$

2. $f(x)$ is a continuous function, (8%)

a. Prove that $\int_0^{\pi} x f(\sin x) dx = \int_0^{\pi} (\pi - x) f(\sin x) dx$

b. Using the result obtained from a., prove that

$$\int_0^{\pi} x f(\sin x) dx = \pi \int_0^{\pi/2} f(\sin x) dx$$

and find the value of

$$\int_0^{\pi} x \sin^2 x dx$$

3. Find the differential for the following: (24%)

a. $x - \frac{1}{x^2}$

b. $\frac{1}{x^3 \sqrt{x}}$

c. $\left(x - \frac{1}{\sqrt{x}}\right)^2$

d. $\sqrt[3]{\frac{x+1}{x+4}}$

e. $\frac{\sqrt{1+x}}{1-x^2}$

f. $\sqrt{\frac{(x-1)(x+3)}{(x+1)^3}}$

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4. Find the value of the following integrals: (40%)

a. $\int_2^9 \frac{dx}{3x^2 - 5x + 2}$

b. $\int_1^{\infty} \frac{1 + \ln x}{x^2} dx$

c. $\int_0^1 \frac{\ln x}{x} dx$

d. $\int_0^{\infty} x e^{-x^2} dx$

e. $\int_1^9 \frac{x dx}{\sqrt{x-1}}$

f. $\int_{-\infty}^{\infty} \frac{dx}{x^2 + 2x + 2}$

g. $\int_{\frac{\pi}{2}}^{\pi} e^{2x} \cos 3x dx$

h. $\int_0^{\pi} x^3 \sin x dx$

i. $\int_1^{\infty} \frac{dx}{x^2(x+1)}$

j. $\int_0^{\infty} \frac{\sin^2 x}{x^2} dx$

5. Find the maximum and minimum for the following function, assuming that (8%)

$$a > 0, \text{ and } f(x) = x - \sqrt{2ax - x^2}$$