八十六學年度 生命科學 系（所） 分生組丙 組碩士班研究生入學考試

科目 微積分 科號 [101] 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

1. Evaluate the following integrals: (20%)

(a) $\int_0^{\pi/2} \frac{a\,dx}{(x-a)(x-2a)}$

(b) $\int \left(e^x - 1\right)^4 e^x\,dx$

(c) $\int \frac{dx}{1+\sin x+\cos x}$

(d) $\int x^7(\sin x+\cos x)\,dx$

2. Evaluate the following differentials: (20%)

(a) $y=\sqrt{x^3+2x+1}$ (b) $y=\ln\tan x$

(c) $y=\sinh^3 x$ (d) $y=\frac{1+x^2}{x\sqrt{1-x^2}}$

3. Find the limit of the followings: (10%)

(a) $\lim_{x\to 0}\frac{\sin 2x}{x}$ (b) $\lim_{x\to 0}\frac{\tan x-\sin x}{x^3}$

4. Prove that $\lim_{x\to 0} f(x)$ is not exist if $f(x)=\frac{x^2-x}{|x|}$ (5%)

5.Prove the following inequalities: (10%)

(a) When 0< x <1; $\sqrt{1-x^7}<\sqrt{1-x^8}<\sqrt{2}\sqrt{1-x^2}$

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(b) $\pi/4 < \int_0^{\sqrt{2}/2} \sqrt{1-x^4}\,dx < \frac{\sqrt{2}}{4}\pi$

6. Find the maximum and minimum values for the function $y = x(\log x)^2$ and draw a corresponding cure for the function. (15%)

7. Suppose there is a curve, $x = \tan\theta$, $y = \cos 2\theta$, and $-\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$.

(a) find the coordinates of the intersection points that the curve makes with the x-axis. (10%)

(b) find the volume of a solid body that is generated by rotating the curve around the x-axis. (10%)