1. Find the ratio of the areas of two parts of a circle $x^2 + y^2 = 8$ that is bisected by a parabola $y^2 = 2x$. (10%)

2. Find the sum of the following: (10%)

$$\frac{1}{\ln 2} + \frac{1}{\ln^2 3} + \cdots + \frac{1}{\ln^4(n+1)} + \cdots$$

3. Prove the following equalities: (20%)

(a) $\int_0^{\pi} x f(\sin x)\,dx = \pi \int_0^{\pi/2} f(\sin x)\,dx$

(b) $\int_{-1}^{1} \frac{x^7 + 2x^5 + 8x}{\cos^4 x + 1}\,dx = 0$

(c) $\int_{-\pi/2}^{\pi/2} (x^4 + x^2 - 3)\sin^3 2x\,dx = 0$

(d) $\int_{-3}^{3} e^{\cosh x}\,dx = 2\int_0^{7} e^{\cosh x}\,dx$

4. Find the limits of the following: (10%)

(a) $\lim_{x \to 4} \left(\frac{2x^2 + x - 2}{x^2 + 3x + 6}\right)^{\frac{1}{x-4}}$

(b) $\lim_{x \to 0} \frac{(1+2x)^{1/2} - (1+3x)^{1/3}}{(1+4x)^{1/4} - (1+5x)^{1/5}}$

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5. Evaluate the following integrals: (40%)

(a) $\int \frac{dx}{(x+2)\sqrt{x^2+4x-5}}$

(b) $\int \frac{4x\,dx}{\quad - x^4}$

(c) $\int_0^1 \frac{x^3-7}{x^3-8}dx$

(d) $\int \frac{dx}{a+b\cos x}$

(e) $\int e^x \cos x\,dx$

(f) $\int x e^{2x}\,dx$

(g) $\int \log(2x+1)\,dx$

(h) $\int \frac{x^2+x}{x^2-x+1}dx$

6. Assume that f(x) is a differentiable function and f(1)=6. The function can also satisfy the following equality as long as x>0,

$$xf(x) - \int_0^x f(t)\,dt = x^2 + 2x^2$$

What is f(x) ? (10%)