國 立 清 華 大 學 命 題 紙

八十四學年度 生命科學 所 乙 組碩士班研究生入學考試

科目 微積分 科號 1001 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

I、填充題(每一空格六分，請將答案依甲，乙，丙，丁・・次序寫出，不需演算過程)

1. The line normal to the plane curve $2x + \sin y = xy$ at $(2, 0)$ is 甲.

2. If $f(x) = x^3 + ax^2 + bx$ has a local minimum at $x = 4$ and a point of inflection at $x = 1$ then $a =$ 乙 and $b =$ 丙.

3. Evaluate
   (a) $\lim_{x \to 1^+} (\frac{1}{x-1} - \frac{1}{\ln x}) =$ 丁.

   (b) $\int_2^4 \frac{\sqrt{x^2-4}}{x} dx =$ 戊.

   (c) $\int_1^\infty \frac{\ln x}{x^2} dx =$ 己.

4. A spherical ballon is inflated with helium at the rate of $100\ \pi$ ft^3/min. How fast is the surface area changing at the instant the radius is 5 ft? 庚.

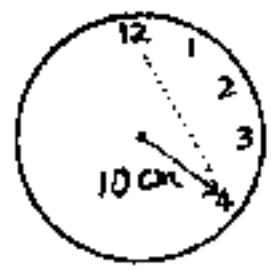
5. The velocity of a body moving along a line from $t = 0$ to $t = \frac{3}{2}$ sec was $v(t) = 5 \cos \pi\ t$ m/sec. Then the total distance traveled is 辛.

6. If $\sqrt{x^2 + 2x + 3} = a_0 + a_1(x+1) + a_2(x+1)^2 + a_3(x+1)^3 + a_4(x+1)^4 + \cdots$ is the Taylor's expansion, then $a_4 =$ 壬.

7. Let R be the semicircular region bounded by the x-axis and the semi-circle $y = \sqrt{1 - x^2}$. Evaluate $\iint_R e^{x^2+y^2} dxdy =$ 癸.

II、計算與証明題(每一大題十分，必需寫出演算証明過程)

1. At what rate is the distance between the tip of the second hand and the 12 o'clock mark changing when the second points to 4 o'clock?



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2. Show that $1 - \frac{\pi^2}{4^2 2!} + \frac{\pi^4}{4^4 4!} - \cdots + \frac{(-1)^k(\pi)^{2k}}{4^{2k}(2k)!} + \cdots$ is convergent and the sum is $\frac{1}{\sqrt{2}}$.

3. Find the volume of the region common to the interiors of the cyclinders $x^2 + y^2 = 1$ and $x^2 + z^2 = 1$.

4. Find the maximum value of $f(x, y, z) = 8x^2 - 4yz - 16z + 600$ on the ellipsoid $4x^2 + y^2 + 4z^2 = 16$.