**注意：考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。**

國立清華大學 108 學年度碩士班考試入學試題

系所班組別：生命科學院 丙組

考試科目(代碼)：計算機概論(演算法與計算機數學)(0604)

## —作答注意事項—

1. 請核對答案卷（卡）上之准考證號、科目名稱是否正確。
2. 作答中如有發現試題印刷不清，得舉手請監試人員處理，但不得要求解釋題意。
3. 考生限在答案卷上標記「由此開始作答」區內作答，且不可書寫姓名、准考證號或與作答無關之其他文字或符號。
4. 答案卷用盡不得要求加頁。
5. 答案卷可用任何書寫工具作答，惟為方便閱卷辨識，請儘量使用藍色或黑色書寫；答案卡限用 2B 鉛筆畫記；如畫記不清（含未依範例畫記）致光學閱讀機無法辨識答案者，其後果一律由考生自行負責。
6. 其他應考規則、違規處理及扣分方式，請自行詳閱准考證明上「**國立清華大學試場規則及違規處理辦法**」，無法因本試題封面作答注意事項中未列明而稱未知悉。

# 國立清華大學 108 學年度碩士班考試入學試題

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**1.** (20%) Let A be a real matrix with n rows and n columns. State or explain what the following algorithms do.

(a) (5%) LU-decomposition for A.

(b) (5%) QR-factorization for A.

(c) (5%) Singular value decomposition for A.

(d) (5%) Spectrum decomposition if A is real and symmetric.

**2.** (15%) Let an n-tuple column vector $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ be denoted as $\vec{x} \in R^n$, a Householder matrix $H_u$ can be defined as $H_u = \mathrm{I} - 2\vec{u}\vec{u}^t$, where I is an identity matrix and $\vec{u} \in R^n$ with $\| \vec{u} \|_2 = 1$. An example is: let $\vec{u} = \begin{bmatrix} 1/2 \\ \sqrt{3}/2 \end{bmatrix}$, then $H_u = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - 2\begin{bmatrix} 1/2 \\ \sqrt{3}/2 \end{bmatrix}\begin{bmatrix} 1/2 & \sqrt{3}/2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$.

Show that

(a) (5%) A Householder matrix is symmetric.

(b) (5%) The inverse of a Householder matrix is itself.

(c) (5%) A Householder matrix is orthogonal.

**3.** (15%) Let $A = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ and let $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in R^3$.

(a) (5%) Find the eigenvalues of A.

(b) (5%) Find the spectrum decomposition for A.

(c) (5%) Find the maximal value for $\vec{x}^t A\vec{x}$ if $\| \vec{x} \|_2 = 1$.

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**4.** (15%) Answer the following questions.

(a) (3%) $\sum_{k=1}^{n}(2n-1)$ = ?

(b) (3%) $\sum_{k=0}^{n}\binom{n}{k}$ = ?

(c) (3%) $\sum_{k=0}^{n}(-1)^k\binom{n}{k}$ = ?

(d) (3%) Define $\Gamma(x)=\int_0^\infty e^{-t}t^{x-1}dt \quad for\ x>0$, compute $\Gamma(6)$.

(e) (3%) Compute $\Gamma\left(\frac{1}{2}\right)$, *where* $\Gamma(x)=\int_0^\infty e^{-t}t^{x-1}dt$.

**5.** (15%) Let $X_1$, $X_1,\cdots$, $X_n$ be a random sample of Bernoulli trials with probability P($X_1=1$)=p, and P($X_1=0$)=q, where 0<p, q<1, and p+q=1.

Then $X=X_1+X_2+\cdots+X_n$ has a binomial distribution.

(a) (5%) What is the probability density function for X?

(b) (5%) What is the moment-generating function for X?

(c) (5%) What are the expectation E(X) and variance Var(X) of X?

**6.** (10%) Let X have a normal distribution with mean 3 and variance 4, define Z=(X-3)/2, and $Y=Z^2$.

(a) (3%) Write down the probability density function of X.

(b) (3%) Write down the probability density function of Z.

(c) (4%) Write down the probability density function of Y.

**7.** (10%) Let $X_{(1)}< X_{(2)}<\cdots< X_{(n)}$ be the order statistic of a random sample $\{X_1, X_2, \cdots, X_n\}$ from the uniform distribution U(0,1).

(a) (5%) Find the probability density function of $X_{(n)}$.

(b) (5%) Use the results of (a) to find the expectation E($X_{(n)}$).