

# 國立清華大學 104 學年度碩士班考試入學試題

系所班組別：生命科學院丙組

考試科目（代碼）：計算機概論(演算法與計算機數學)(0604)

共\_\_3\_\_頁，第\_\_1\_\_頁 \*請在【答案卷】作答

1. a) (5%) How many solutions are there for the equation  
 $x_1 + x_2 + x_3 = 11$  where  $x_1, x_2$  and  $x_3$  are non-negative integers?  
b) (5%) How about when  $x_1 \geq 1, x_2 \geq 2, x_3 \geq 3$ ?
2. (10%) What is the solution of the recurrent relation  
 $a_n = a_{n-1} + 2a_{n-2}$  with  $a_0 = 2$  and  $a_1 = 7$ ?
3. (15%) Let  $C[-\pi, \pi]$  with inner product  
$$\langle f, g \rangle = \int_{-\pi}^{\pi} f(x)g(x)dx.$$

Supposed  $\{1, \cos x, \sin x\}$  is an orthogonal set of vectors for the inner product space  
(e.g.  $\langle 1, \cos x \rangle = \langle 1, \sin x \rangle = \langle \cos x, \sin x \rangle = 0$ ).

Find an orthonormal set of the vectors for the space.
4. a) (7%) Compute the eigenvalues of matrix  $A = \begin{bmatrix} 2 & -3 \\ 2 & -5 \end{bmatrix}$   
b) (8%) Find the matrix  $S$  such that  $S^{-1}AS$  is a diagonal matrix
5. (10 %) Suppose that 5 men out of 100 and 25 women out of 10,000 are color blind. A color blind person is chosen at random. What is the probability of his being male? (Assume males and females be equal numbers.)

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共 3 頁，第 2 頁 \*請在【答案卷】作答

6. a) (7%) The Heapify algorithm makes an array  $A$  of elements under node  $i$  satisfy the heap property that is given as following:

**Heapify**( $A, i$ )

```
1   $l \leftarrow \text{Left}(i)$ 
2   $r \leftarrow \text{Right}(i)$ 
3  if  $l \leq \text{heap-size}[A]$  and  $A[l] > A[i]$ 
4    then  $\text{largest} \leftarrow l$ 
5    else  $\text{largest} \leftarrow i$ 
6  if  $r \leq \text{heap-size}[A]$  and  $A[r] > A[\text{largest}]$ 
7    then  $\text{largest} \leftarrow r$ 
8  if  $\text{largest} \neq i$ 
9    then exchange  $A[i] \leftrightarrow A[\text{largest}]$ 
10 Heapify( $A, \text{largest}$ )
```

where  $\text{Left}(i)$  and  $\text{Right}(i)$  returns indices of the left and right children of node  $i$  respectively.

How many interchange operations are needed to  $\text{Heapify}(A, 3)$  on the array  $A = \langle 27, 17, 3, 16, 13, 10, 1, 5, 7, 12, 4, 8, 9, 0 \rangle$ ?

- b) (10%) Suppose  $A$  is an array that satisfies the heap property. Using the above heapify algorithm to design a Heapsort algorithm to sort the elements in  $A$  as following:

**Heapsort**( $A$ )

```
1 for  $i \leftarrow \text{length}[A]$  downto 2
2   do exchange  $A[1] \leftrightarrow A[i]$ 
3    $\text{heap-size}[A] \leftarrow \text{heap-size}[A] - 1$ 
4   Heapify( $A, 1$ )
```

What is the time complexity of the Heapsort algorithm in terms of number of elements  $n$ . Justify your answer.

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共\_\_3\_\_頁，第\_\_3\_\_頁 \*請在【答案卷】作答

7. a) (7%) In Quick-sort of  $n$  distinct elements? There is an partition procedure that partitions a subarray of  $A$  from  $p$  to  $q$ ,  $[A, p, q]$  as following:

```
PARTITION(A,p,r)
1  x ← A[p]
2  i ← p - 1
3  j ← r + 1
4  while True
5      do repeat j ← j - 1
6          until A[j] ≤ x
7          repeat i ← i + 1
8          until A[i] ≥ x
9  if i < j
10     then exchange A[i] ↔ A[j]
11     else return j
```

Illustrate the operations of PARTITION algorithm on the input array

$A = \langle 13, 19, 9, 5, 12, 8, 7, 4, 11, 2, 6, 21 \rangle$

- b) (8%) What is the worst case running time complexity of partitioning in terms of  $n$  elements in array  $A$ ?
- c) (8%) What is the best case running time complexity of partitioning in terms of  $n$  elements in array  $A$ ?