國立清華大學 103 學年度碩士班考試入學試題

系所班組別：生命科學院丙組

考試科目（代碼）：計算機概論(演算法與計算機數學)(0604)

1. (14%) Find the asymptotic bounds for the recurrences T(n):

   a) $T(n) = T(n-1) + n$

   b) $T(n) = 3\ T(n/3) + n \log n$

2. (14%) application of number theory

   a) Use Fermat's little theorem to compute 5^{2003} mod 7, 5^{2003} mod 11, and 5^{2003}mod 13,

   b) Use your results from part a) and the Chinese Remainder Theorem to find 5^{2003}mod 1001.

3. (15%) Solve the simultaneous recurrent relations

   $a_n = 3\ a_{n-1} + 2b_{n-1}$

   $b_n = a_{n-1} + 2\ b_{n-1}$

   while the initial conditions are a_0=1, b_0=2 respectively.

4. (7%) The probability that a driver will have an accident in 1 month equals to 0.02. Assume accident occurrence be a Poisson process, find the probability p that in 100 months he will have 3 accidents.

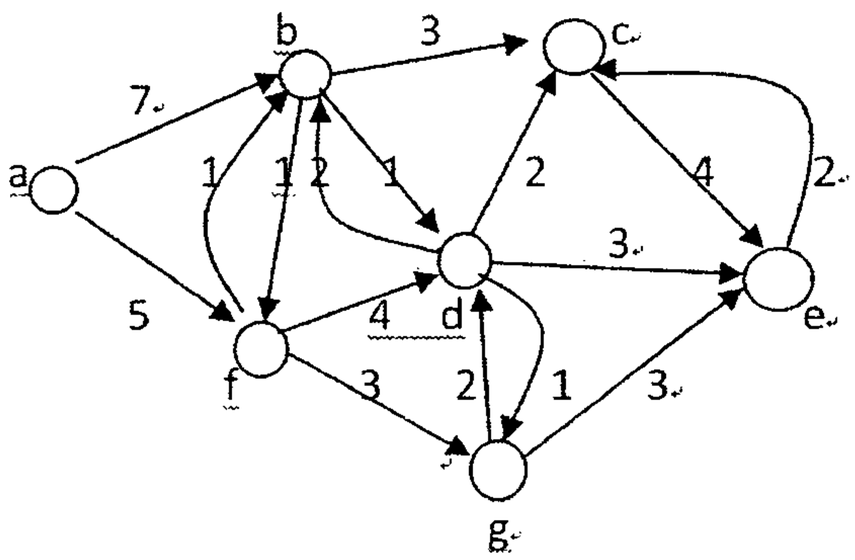
5. Consider the biodynamic system

$$X(t+1) = \begin{bmatrix} 3 & -5 \\ 1 & -1 \end{bmatrix} X(t)$$

   a) (10%) Find real closed formula for each component of the two-dimensional vector $X(t)$.

   b) (5%) Is the system stable? Please justify your answer.

6. (a) (10%) Run Dijkstra's algorithm to find the shortest path to each node in the following weighted directed graph G(V, E) starting from the source node a, the numbers attached to the edges represent the distance weight. Please show the node sequence in which each node's shortest path is found by the algorithm.



(b) (5%) In Dijkstra's algorithm, EXTRACT-min operation is used to find the shortest distance node in the priority queue. What is the time complexity of Dijkstra's algorithm in terms of numbers of nodes V and edges E if EXTRACT-min takes O(V)?

7. (a) (5%) Construct a finite state automaton to identify the substring pattern of 'tataag' in a long string of a nuclear acid sequence.

(b) (15%) Let $\pi[q]$ be the prefix function that calculates the length of the longest prefix of P that is a proper suffix of P_q.

$$\pi[q] = \max\{k: k < q \text{ and } P_k \leqq P_q\},$$

where $P_k \leqq P_q$ means P_k is a prefix string of P_q.

Use Prefix function π in designing Knuth-Morris-Pratt algorithm for string matching of pattern P in a long string T, e.g. KMP(T, P). Show also the time complexity of KMP algorithm if the string length of T and P are n and m respectively.