科目：近代物理(500G)

校系所組：交通大學電子研究所（甲組）
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Part I: 1~8 題為單一選擇題(每一題 5 分，共 40 分，每題答錯倒扣 1.25 分。)

Part II: 9~16 題為多重選擇題(每一題 5 分，共 40 分，單一選項答錯倒扣 1 分。)

Part III: 17 題為非選擇題(共 20 分)

Some physical constants and integrals:

1. Planck's constant $h = 4.16\times10^{-15}$ eV$\cdot$s $= 6.62\times10^{-34}$ joul$\cdot$s
2. Reduced Planck's constant $\hbar = h/2\pi = 1.05\times10^{-34}$ joul$\cdot$s
3. Elementary charge of an electron $e = -1.6\times10^{-19}$ C
4. Electron mass $m_e = 9.1\times10^{-31}$ kg
5. $\int_0^\infty x\exp(-ax)dx = \frac{1}{a^2}$
6. $\int_0^\infty x^2\exp(-ax^2)dx = \frac{1}{4a}\sqrt{\frac{\pi}{a}}$
7. Laplacian operator in spherical coordinate $\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}$

**Part I 單一選擇題**

1. Density of state function $g(E)$ is defined as the number of available per unit energy interval. The $g(E)$ of one-dimension, two-dimension, and three-dimension free electron gas model is proportional to
   (A) $E^1, E^2, E^3$, (B) $E^3, E^2, E^1$, (C) $E^0, E^1, E^2$, (D) $E^{3/2}, E^1, E^{1/2}$, (E) $E^{-1/2}, E^0, E^{1/2}$
2. Considering the free electrons system in a metal with a volume $V$, total number of electrons is $N$, for $T = 0$, the Fermi energy $E_F$ is
   (A) $\frac{h^2}{2m}(\frac{3N}{8\pi V})^{1/2}$, (B) $\frac{h^2}{2m}(\frac{3N}{8\pi V})^{1}$, (C) $\frac{h^2}{2m}(\frac{3N}{8\pi V})^{3/2}$,(D) $\frac{h^2}{2m}(\frac{3N}{8\pi V})^{1/3}$,(E) $\frac{h^2}{2m}(\frac{3N}{8\pi V})^{2/3}$
3. Consider two identical fermions, each in the same one-dimensional infinite well with length $L$. The single-particle wave function in that case is $u_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)$, where $n$ is the quantum number that labels the state. Disregard the normalization constant, which of the following expressions is the correct wave function of the two-fermion system?
   (A) $\sin^2\left(\frac{n\pi x}{L}\right)$, (B) $\sin\left(\frac{n\pi x_1}{L}\right)\cos\left(\frac{m\pi x_2}{L}\right)$, (C) $\sin\left(\frac{n\pi x_1}{L}\right)+\cos\left(\frac{m\pi x_2}{L}\right)$, (D) $\sin\left(\frac{n\pi x_1}{L}\right)\sin\left(\frac{m\pi x_2}{L}\right)$, (E) Non of the above
4. Which is the correct ground-state $L$ and $S$ of fluorine?
   (A) $L$=1, $S$=1/2, (B) $L$=2, $S$=0, (C) $L$=0, $S$=3/2, (D) $L$=2, $S$=1/2, (E) $L$=1, $S$=1
5. Please calculate and indicate the energy level of the first excited state for the hydrogen atom.
   (A) -13.6 eV, (B) -10.2 eV, (C) -6.8 eV, (D) -12.1 eV, (E) -3.4 eV
6. As an α particle of mass $m_0$ with a speed $V$ strikes a stationary proton with a mass $\frac{1}{4}m_0$, please find the percent change in velocity for the α particle after the collision.
   (A) −20%, (B) +20%, (C) −40%, (D) −80%, (E) 0%

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7. Stefan law for the total radiation emitted by a black body at all wavelengths is
(A) $5.67\times10^{-8}T^4$ W・m$^{-2}$ ・k$^{-4}$(B) $5.67\times10^{-8}T^2$ W・m$^{-2}$ ・k$^{-4}$,
(C) $5.67\times10^{-8}\exp(-\frac{1}{KT})$ W・m$^{-2}$ ・k$^{-4}$, (D) $5.67\times10^{-8}T$ W・m$^{-2}$ ・k$^{-4}$,
(E) $5.67\times10^{-8}T^3$ W・m$^{-2}$ ・k$^{-4}$, where $T$ is the absolute temperature of the black body.

8. As the X-ray is collided elastically with a free electron, the scattered X-ray will have a wavelength of
(A) $\lambda_0 - \frac{h}{m_0C}(1-\cos\theta)$, (B) $\lambda_0 + \frac{h}{m_0C}(1+\cos\theta)$, (C) $\lambda_0 + \frac{h}{m_0C}(1-\cos\theta)$,
(D) $\lambda_0 + \frac{h}{m_0C}(1-\sin\theta)$, (E) $\lambda_0 + \frac{h}{m_0C}(1+\sin\theta)$
Where $\lambda_0$ is the wavelength of the X-ray before collision, $h$ is Planck's constant, $m_0$ is the rest mass of the electron, $C$ is the speed of light, and $\theta$ is the scattering angle between the incident X-ray and the scattered X-ray.

**Part Ⅱ多重選擇題**

9. For specific heats of solids, which of following statements are "true"?
($C_V$ is defined as specific heats at constant volume, $N$ is the number of atoms in solid, $k$ is Boltzmann constant)
(A) In classical Dulong-Petit law, $C_V = (3/2)Nk$.
(B) In Einstein's model, average energy per oscillator is $\langle E\rangle = (1/2)kT$.
(C) In Einstein's model, each oscillator vibration is independent.
(D) In Debye model, each oscillator vibration is independent.
(E) In Debye model, phonons obey Bose-Einstein distribution function.

10. For a Si semiconductor material, which of following statements are "true"?
(A) Si is ionic bond,
(B) For an isolated P-type Si semiconductor bulk, the system is with net "positive" charge.
(C) For an isolated N-type Si semiconductor bulk, the system is with net "negative" charge.
(D) For a Si PN diode under thermal equilibrium without bias, the Fermi level $E_F$ is the same ($E_{Fn} = E_{Fp}$).
(E) For a Si PN diode under forward bias, the N-type Fermi level $E_{Fn}$ is higher than P-type Fermi level $E_{Fp}$ ($E_{Fn} - E_{Fp} > 0$).

11. The three-dimensional Schrodinger equation can be solved for the case of a coulomb potential energy--the case of the atom consisting of a single electron orbiting a nucleus. Which of the following statements are correct?
(A) For the central coulomb potential, angular momentum is quantized in integer multiples of Planck's constant, with a coefficient depending on an angular momentum quantum number $l$.
(B) The energy eigenvalues, which depend on a principal quantum number, are those of the Bohr model.
(C) The energies are independent of all the possible angular-momentum values, so that there is a significant degree of degeneracy.
(D) Only an electron in a state with nonvanishing angular momentum forms a current loop, or equivalently, a magnetic moment that can be measured by looking at shifts in atomic energy levels in an external magnetic field.
(E) An electron interacts with the magnetic field it experiences as it moves in a coulomb field. This interaction breaks the degeneracy of the atomic levels even without an external field.

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12. An electron is described by the wave function

$$\psi(x)=\begin{cases} 0 & for \quad x<0 \\ Ae^{-x}(2-e^{-x}) & for \quad x\ge 0 \end{cases}$$

where $x$ is in nanometers and $A$ is a constant. Which of the following statements are correct?

(A) $A=\sqrt{\frac{12}{11}}$,

(B) The electron most likely to be found is nothing to do with the value of $A$.

(C) The electron most likely to be found is at $x=0.52$ nm.

(D) The expectation value of $x$ for this electron is 0.67 nm.

(E) The position where the electron most likely be found is not necessarily the same with the expectation value of on $x$ for the electron.

13. For the special relativity, which items in the following statements are wrong?

(A) The observers in different inertial frames always measure different time intervals between a pair of events.

(B) The proper length of an object is defined as the length of the object measured by someone who is at rest with respect to the object and the length measured for the relative moving reference frame is always longer than the proper length.

(C) The total energy E is expressed as $E=\frac{MC^2}{\sqrt{1-\frac{V^2}{C^2}}}$ and $E=P^2C^2-(MC^2)^2$， where $M$ is the mass of the particle and $V$ is the particle velocity as well as the $P$ is the particle momentum and $C$ is the speed of light.

(D) Planck's law correctly explains the quantum of the energy for the black body.

(E) The Compton effect proves that the X-ray photons behave like waves.

14. As an electron with an accelerating voltage of thousands of volts $V$ collides with a metal target, it will produce an X-ray (A) with a wavelength $\lambda=\frac{hC}{eV}$, (B) with a wavelength $\lambda=\frac{hC^2}{eV}$, (C) with a wavelength $\lambda=\frac{hC^2}{meV}$, (D) with a broad continuous spectrum plus a number of sharp lines, (E) with only a number of sharp lines for the spectrum, where $h$ is Planck's constant, $C$ is the speed of light, and $m$ is the mass of the electron.

15. The Einstein's theory of the photoelectric effect explains that

(A) the different metals have different work functions.

(B) the light has a minimum threshold frequency to emit the photoelectron.

(C) the photocurrent is zero as the frequency of light is less than the threshold frequency.

(D) the photocurrent is higher as if the light has a higher frequency.

(E) the emitted electrons have the kinetic energy linearly proportional to the frequency of light.

16. The spectral lines for hydrogen

(A) have four for the visible light.

(B) have five for the visible light.

(C)have four for the ultraviolet （UV） light.

(D) have visible, UV, and infrared（IR） lights.

(E) have no IR light.

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**Part III 非選擇題**

17. The electron in hydrogen occupies the $n=2$ and $l=1$ quantum state.

(A) Use the given Table I and II to find the most likely distance from the origin for the electron? (5%)

(B) Verify that the eigenvalue $E_2$ of the wave function that has highest probability density in the plane parallel to the $z$ axis satisfies the time-independent Schrodinger equation $-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi$. (Bohr radius $a_0 = \frac{4\pi\varepsilon_0\hbar^2}{me^2}$) (10%)

(C) Schematically plot the probability density that the electron may have. (5%)

Table I: Spherical harmonics

| $\ell, m_\ell$ | $\Theta_{\ell,m_\ell}(\theta)\Phi_{m_\ell}(\phi)$ |
|---|---|
| 0, 0 | $\sqrt{\frac{1}{4\pi}}$ |
| 1, 0 | $\sqrt{\frac{3}{4\pi}}\cos\theta$ |
| 1, ±1 | $\sqrt{\frac{3}{8\pi}}\sin\theta e^{\pm i\phi}$ |
| 2, 0 | $\sqrt{\frac{5}{16\pi}}(3\cos^2\theta - 1)$ |
| 2, ±1 | $\sqrt{\frac{15}{8\pi}}\cos\theta\sin\theta e^{\pm i\phi}$ |
| 2, ±2 | $\sqrt{\frac{15}{32\pi}}\sin^2\theta e^{\pm 2i\phi}$ |
| 3, 0 | $\sqrt{\frac{7}{16\pi}}(5\cos^3\theta - 3\cos\theta)$ |
| 3, ±1 | $\sqrt{\frac{21}{64\pi}}(5\cos^2\theta - 1)\sin\theta\, e^{\pm i\phi}$ |
| 3, ±2 | $\sqrt{\frac{105}{32\pi}}\cos\theta\sin^2\theta\, e^{\pm 2i\phi}$ |
| 3, ±3 | $\sqrt{\frac{35}{64\pi}}\sin^3\theta\, e^{\pm 3i\phi}$ |

Table II: Radial wave function

| $n, \ell$ | $R_{n,\ell}(r)$ |
|---|---|
| 1, 0 | $\frac{1}{(1a_0)^{3/2}}2e^{-r/a_0}$ |
| 2, 0 | $\frac{1}{(2a_0)^{3/2}}2\left(1-\frac{r}{2a_0}\right)e^{-r/2a_0}$ |
| 2, 1 | $\frac{1}{(2a_0)^{3/2}}\frac{r}{\sqrt{3}a_0}e^{-r/2a_0}$ |
| 3, 0 | $\frac{1}{(3a_0)^{3/2}}\left(2-\frac{4r}{3a_0}+\frac{4r^2}{27a_0^2}\right)e^{-r/3a_0}$ |
| 3, 1 | $\frac{1}{(3a_0)^{3/2}}\frac{4\sqrt{2}r}{9a_0}\left(1-\frac{r}{6a_0}\right)e^{-r/3a_0}$ |
| 3, 2 | $\frac{1}{(3a_0)^{3/2}}\frac{2\sqrt{2}r^2}{27\sqrt{5}a_0^2}e^{-r/3a_0}$ |