1. Consider a wave incident upon a convex lens of focal length ***R***, thickness ***d***, and refractive index ***n***, as shown in the following figure. The portion of the wavefront near the axis of the lens travels through a thicker section of the optically dense material of the lens than the portion farther away from the axis.

   (a) Derive the path difference for the light ray **(1)**, through the axis of the lens, and the light ray **(2)**, with a distance $x0$ away from the axis. (5%)

   (b) If the lens is thin, $x0 << R$, show that the lens introduces a thickness-independent phase delay. (5%)

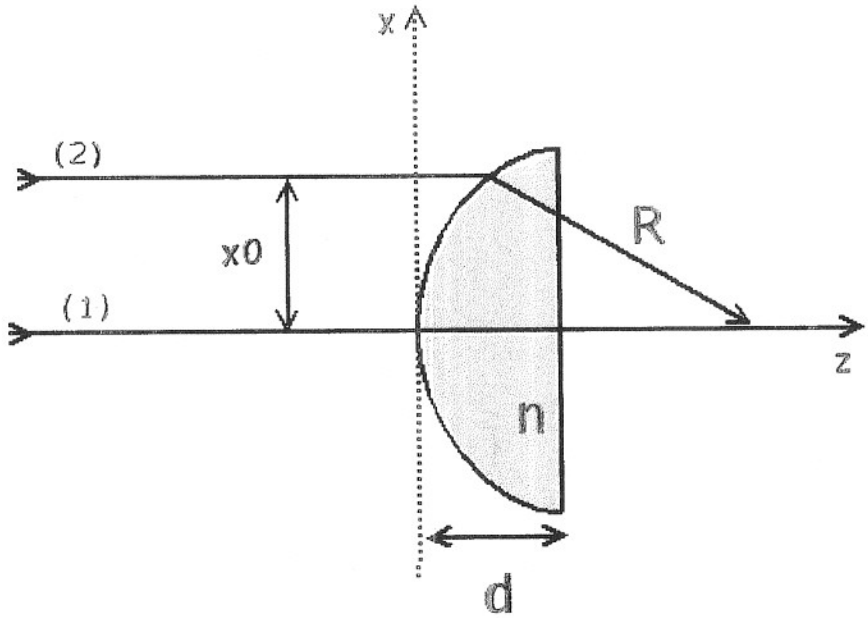


Fig. 1

2. A glass rod of rectangular cross-section is bent into the shape of ***U***, shown in the Fig. 2(a). A parallel beam of light falls perpendicularly on the flat surface ***A***. Determine the minimum value of the ratio ***R/d*** for which all light entering the glass through surface ***A*** will emerge from the glass through surface ***B***. The index of refraction of the glass is ***1.5***. Hit: follow the ray trajectory shown in the Fig. 2(b). (10%)

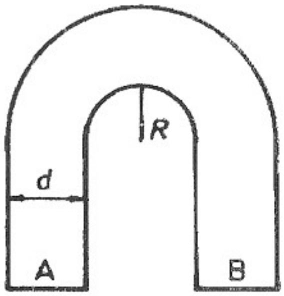


Fig. 2(a)

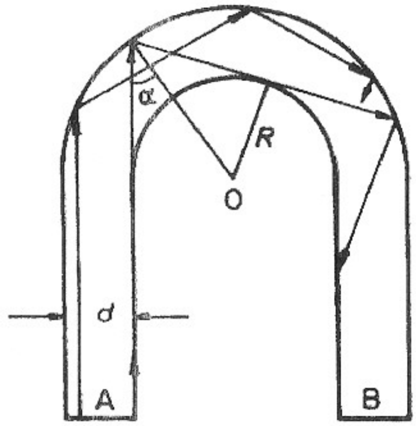


Fig. 2(b)

3. (a) Show that $E = f_+(z - vt) + f_-(z + vt)$ is the solution for one-dimensional wave equation, $\nabla^2 E - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}E = 0$, where $\nabla^2 = \frac{\partial^2}{\partial x^2}$. (3%)

(b) What is the difference between the components $f_+(z - vt)$ and $f_-(z + vt)$? (2%)

(c) In the spherical coordinates, $\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial}{\partial r}) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial}{\partial\theta}) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}$, write down the solutions for the ***spherical waves***. (5%)

4. The *phase velocity* and *group velocity* of wave are defined as

$$v_p = \frac{\omega}{k} \text{ and } v_g = \frac{d\omega}{dk}.$$

(a) For a dispersive media, derive the following formula (4%),

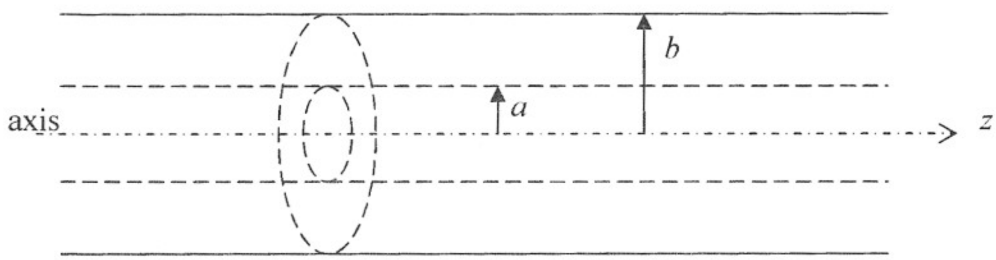
$$v_g = v_p - \lambda\frac{dv_p}{d\lambda}.$$

(b) In terms of the vacuum wavelength, λ_0, the index of refraction, n, and the speed of light, c, derive the following formula (3%),

$$\frac{1}{v_g} = \frac{1}{v_p} - \frac{\lambda_0}{c}\frac{dn}{d\lambda_0}.$$

(c) For a particular type of glass with the index of refraction approximately by an empirical equation, $n = A + B\lambda_0^{-2}$, find the group velocity at $\lambda_0 = 500nm$ for $A = 1.50$ and $B = 3\times10^4(nm)^2$. (3%)

5. A stream of CW electron charges is accelerated from zero velocity to a uniform longitudinal velocity $u\vec{a}_z$ by using a DC electron accelerator, where $\vec{a}_z$ is a unit vector in the z direction. Assume at the exit of the electron accelerator the generated electron beam appears to be a very long CW round beam. The electron beam has a uniform volume charge density ρ within a radius b, as shown below. Let's denote the electron charge by e.



A stream of round electron beam propagating along z

(a) What is the output current of the DC accelerator? (3%)

(b) What is the electric force experienced by an electron at radius a from the beam axis, where $a < b$? Indicate the magnitude and direction of the force. (3%)

(c) What is the magnetic flux density generated by the moving electrons at radius a? (3%)

(d) What is the magnetic force experienced by the electron at radius a? Indicate the magnitude and direction of the force. (3%)

(e) What is the electron velocity that is required to exactly balance the magnetic force and the electric force? Is it possible to achieve this velocity? (3%)

6. The region between two concentric spherical-shell electrodes are filled with two different dielectrics with permittivities $\varepsilon_1, \varepsilon_2$ in regions I and II, respectively, as shown below. The inner electrode has a radius a and is maintained at a voltage V_1, and the outer electrode has a radius b and is maintained at a voltage V_2.

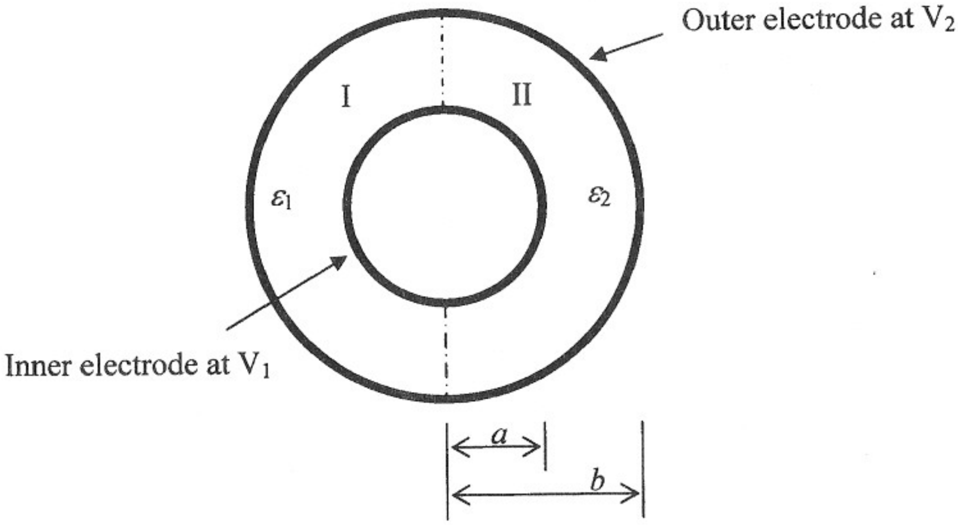
(a) What is the capacitance of this device? (5%)

(b) What are the electric field intensities in regions I and II? Indicate the magnitude and direction of the fields. (2%)

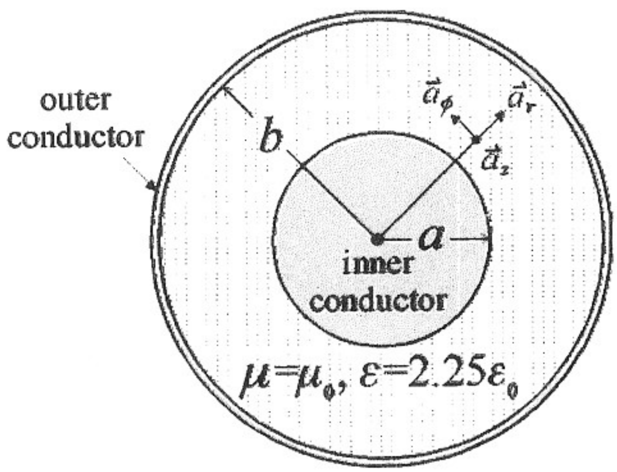
(c) What are the electric flux densities in regions I and II? Indicate the magnitude and direction of the fields. (2%)

(d) What are the surface charge densities induced on the inner electrodes in regions I and II? (2%)

(e) What is the electrostatic energy stored in this device? (4%)



7. A coaxial cable is filled with dielectric material (linear, isotropic, homogeneous, source-free, non-conducting, nonmagnetic, ε_r=2.25) between perfect conductors with inner and outer radii a and b, respectively.



Electrostatics concludes that a constant voltage difference between the inner and outer conductors causes a static E-field $\vec{E}(r)=\vec{a}_r E_0 \dfrac{a}{r}$ (V/m) for $r\in[a,b]$. If we connect a sinusoidal voltage source of angular frequency ω (rad/sec) to the input end of the cable, the excited time-harmonic E-field in the dielectric medium should be: $\vec{E}(r,z,t)=\ \vec{a}_r E_0 \dfrac{a}{r}\cos(\omega t-\beta z)$

(V/m), where E_0 is assumed to be a real constant.

You might need the formulae of vector analysis in cylindrical coordinates (r, ϕ, z):

$$\nabla \times \vec{A} = \vec{a}_r\left(\frac{\partial A_z}{r\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \vec{a}_\phi\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \vec{a}_z\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial \phi}\right];$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\nabla^2 \vec{A} = \vec{a}_r\left(\nabla^2 A_r - \frac{2}{r^2}\frac{\partial A_\phi}{\partial \phi} - \frac{A_r}{r^2}\right) + \vec{a}_\phi\left(\nabla^2 A_\phi + \frac{2}{r^2}\frac{\partial A_r}{\partial \phi} - \frac{A_\phi}{r^2}\right) + \vec{a}_z\left(\nabla^2 A_z\right)$$

(a) (5%) Express the propagation constant β by the material and frequency parameters ε_0, μ_0, and ω. Justify your answer.

(b) (5%) Plot the corresponding magnetic field intensity $\vec{H}(r=a, z, t=0)$ (A/m) for $z\in[0,4\pi/\beta]$, and indicate its amplitude and direction.

(c) (4%) Plot the displacement current density $\vec{J}_d(r=a, z, t=0)$ (A/m^2) for $z\in[0,4\pi/\beta]$, and indicate its amplitude and direction.

(d) (4%) Plot the total current in the inner conductor $I_{in}(z, t=0)$ (A) for $z\in[0,4\pi/\beta]$, and indicate its amplitude.

(e) (4%) Plot the surface charge density of the inner conductor $\rho_s(r=a, z, t=0)$ (C/m^2) for $z\in[0,4\pi/\beta]$, and indicate its amplitude.

(f) (4%) What is the time-averaged power density $\vec{P}_{av}(r, z)$ (W/m^2)?

(A) $\vec{a}_z 2.25\sqrt{\frac{\varepsilon_0}{\mu_0}}E_0^2\left(\frac{a}{r}\right)^2$; (B) $\vec{a}_z 0.75\sqrt{\frac{\mu_0}{\varepsilon_0}}E_0^2\left(\frac{a}{r}\right)^2$; (C) $\vec{a}_r 1.5\sqrt{\frac{\varepsilon_0}{\mu_0}}E_0^2\frac{a}{r}$;

(D) $\vec{a}_r 2.25\sqrt{\frac{\mu_0}{\varepsilon_0}}E_0^2\left(\frac{r}{a}\right)^2$; (E) $\vec{a}_z 0.75\sqrt{\frac{\varepsilon_0}{\mu_0}}E_0^2\left(\frac{a}{r}\right)^2$; (F) $\vec{a}_z 1.5\sqrt{\frac{\varepsilon_0}{\mu_0}}E_0^2\ln\left(\frac{a}{r}\right)$;

(G) $\vec{a}_z 2.25\sqrt{\frac{\varepsilon_0}{\mu_0}}E_0^2\ln\left(\frac{a}{r}\right)$; (H) $\vec{a}_r 0.75\sqrt{\frac{\mu_0}{\varepsilon_0}}E_0^2\ln\left(\frac{r}{a}\right)$; (I) $\vec{a}_z 1.5\sqrt{\frac{\varepsilon_0}{\mu_0}}E_0^2\frac{a}{r}$.

(g) (4%) What is the total time-averaged power transmitted in the +z-direction $P_{\mathrm{tot}}(z)$ (W)?

(A) $0.75\sqrt{\frac{\varepsilon_0}{\mu_0}}a^2\pi E_0^2\cos(\beta z)$; (B) $2.25\sqrt{\frac{\mu_0}{\varepsilon_0}}a^2E_0^2\ln\left(\frac{b}{a}\right)$; (C) $1.5\sqrt{\frac{\mu_0}{\varepsilon_0}}b^2E_0^2\ln\left(\frac{b}{a}\right)$;

(D) $0.75\sqrt{\frac{\varepsilon_0}{\mu_0}}b^2E_0^2\ln\left(\frac{b}{a}\right)$; (E) $2.25\sqrt{\frac{\varepsilon_0}{\mu_0}}a^2\pi E_0^2\ln\left(\frac{b}{a}\right)$; (F) $1.5\sqrt{\frac{\mu_0}{\varepsilon_0}}b^2\pi E_0^2\cos(\beta z)$;

(G) $0.75\sqrt{\frac{\mu_0}{\varepsilon_0}}E_0^2\ln\left(\frac{b}{a}\right)\cos(\beta z)$; (H) $2.25\sqrt{\frac{\varepsilon_0}{\mu_0}}\pi E_0^2\ln\left(\frac{b}{a}\right)$; (I) $1.5\sqrt{\frac{\varepsilon_0}{\mu_0}}a^2\pi E_0^2\ln\left(\frac{b}{a}\right)$.