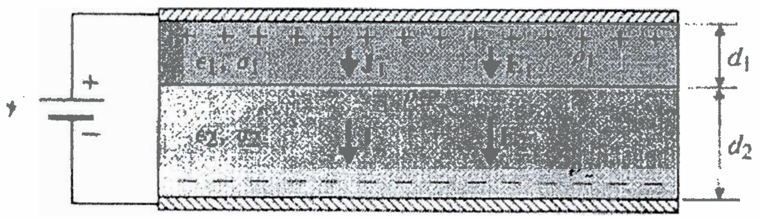
1. (10%) An infinitely long line charge along $-\infty < z < \infty$ has a line charge density of $\rho_l = \alpha|z|$, where α is a constant. The line charge is placed in vacuum.
   (a) What is the electric field at the cylindrical coordinate ($r, \phi, z=0$)? Indicate the magnitude and direction of your answer. (5%)
   (b) What is the energy required to take a charge q from ($r=a, \phi=0, z=0$) to ($r=b, \phi=\pi, z=0$)?(5%)

2. (8%) An electro-motive force V is applied across a parallel-plate capacitor of area S, as shown below. The space between the two conducting plates is filled with two lossy dielectrics of thickness d_1 and d_2, permittivity ε_1 and ε_2, and conductivity σ_1 and σ_2, respectively. Determine
   (a) the resistance of this device, and (4%)
   (b) the capacitance of this device. (4%)

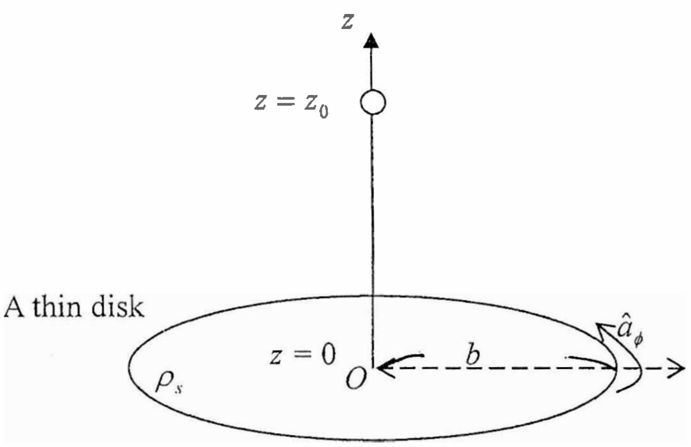


3. (12%) A thin disk of radius b in vacuum contains a uniform, positive surface charge density ρ_s, rotating at an angular frequency of ω about the z axis. The charges on the disk move in the $+\hat{a}_\phi$ direction, as shown below.
   (a) What is the surface current density at a radius r on the disk, where $r < b$? (4%)
   (b) What is the total magnetic dipole moment for this rotating disk? Give the magnitude and direction of your answer. (4%)

(c) What is the magnetic flux density at $z = z_0$? Give the magnitude and direction of your answer. (4%)

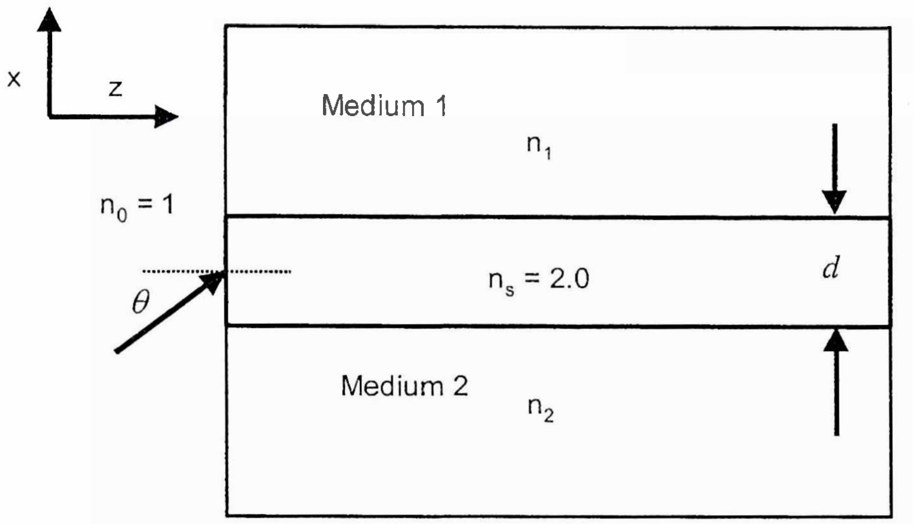
Hints: (i) A current loop with a radius b carrying a current I in vacuum produces an axial magnetic field of $B = \dfrac{\mu_0 I b^2}{2(z^2 + b^2)^{3/2}}$ at a distance z above the current loop, where μ_0 is the vacuum permeability. (ii)

$$\int \frac{x^3 dx}{(x^2 + a^2)^{3/2}} = \sqrt{x^2 + a^2} + \frac{a^2}{\sqrt{x^2 + a^2}}$$



4. (7%) Sea water at frequency $\nu = 4\times10^8$ Hz has permittivity $\varepsilon = 81\varepsilon_0$, permeability $\mu = \mu_0$, and resistivity $\rho = 0.23\Omega$m. What is the ratio of conduction current to displacement current? [Hint: consider a parallel-plate capacitor immersed in sea water and driven by a voltage $V_0 \cos(2\pi\nu t)$.]

5. (23%) A planar slab with refractive index $n_s = 2.0$ is sandwiched between two media with indices n_1 and n_2 respectively. The layer structure is illustrated in the figure below. Suppose that a plane electromagnetic wave with a wavelength 2.0 μm enters the slab at an angle θ. Please answer the following questions.



(a). If the polarization of incident wave is perpendicular to the *x*-*z* plane (TE) and n_1, n_2 = 1.5, what is the maximum angle θ so the wave can be guided in the slab? (2%)

(b). As (a), what is the maximum angle θ if the polarization of incident wave is on the *x*-*z* plane (TM)? (2%)

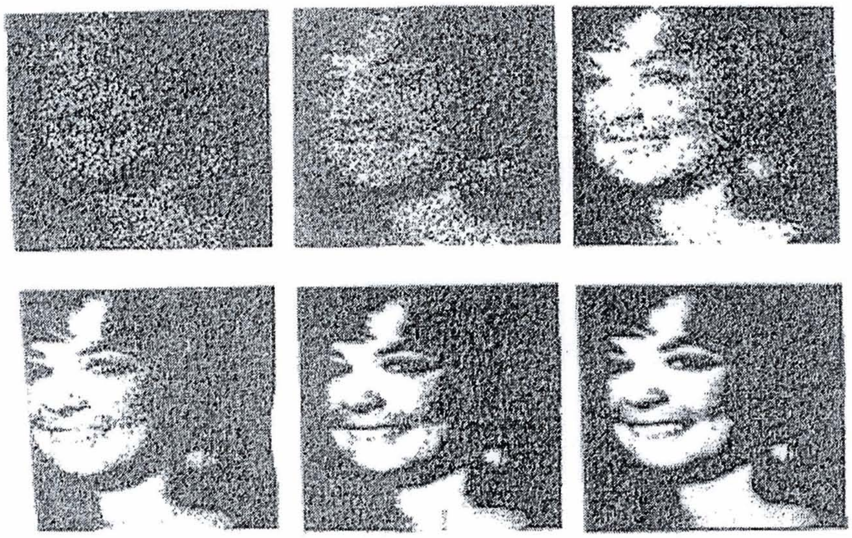
(c). If n_1 = 1.5 and n_2 = 1.3, what is the maximum angle θ so the wave can be guided in the slab? Suppose TE polarization. (2%)

(d). Continue in (c), what is the range of incident angle θ so the wave is leaked to medium 2 but totally reflected at the slab-medium 1 interface? (2%)

(e). Continue in (a). Suppose that the incident angle θ is larger than the maximum guided angle θ_m calculated in (a), what is the power attenuation coefficient (α)? Please express α in terms of θ, θ_m and d. (10%)

(f). Continue in (e). What is the power attenuation coefficient (α) if the incident angle θ is within the range calculated in (e)? (5%)

6. (8 %) The following photographs were taken by controlling the amount of illumination. Using extremely dim light, the first picture (upper left) only begins to suggest an overall image. As the light intensity goes up, the image becomes increasingly recognizable (from left to right in the upper row and then from the left to right in the lower row). What can you conclude about the behavior of light from the photograph demonstration. (Note: The answer is quite simple and just use a few words.)

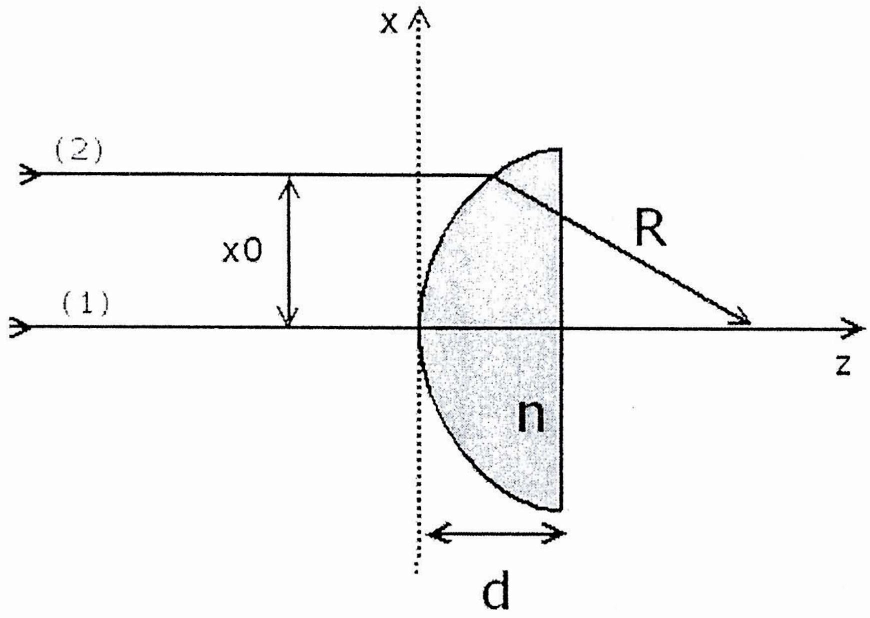


7. (12%) Write mathematical expressions for (a) a spherical wave (b) a plane wave (c) and a cylindrical wave. (No need to derive; only write the expressions.)

8. (7%) Consider a wave incident upon a convex lens of focal length R, thickness d, and refractive index n, as shown in the following figure. The portion of the wavefront near the axis of the lens travels through a thicker section of the optically dense material of the lens than the portion farther away from the axis.

(a) Derive the path difference for the light ray (1), through the axis of the lens, and the light ray (2), with a distance $x0$ away from the axis. (4%)

(b) If the lens is thin, $x0 << R$, show that the lens introduces a thickness-independent phase delay. (3%)



9. (6%) It's well known that white light radiated from the sun has different colors, RED, YELLOW, GREEN, BLUE, etc., but the sky changes its color as the time goes by.

   (a) Explain why the sky is almost in BLUE color in the daytime. (2%)

   (b) Explain why the sky is almost in RED color at sunrise and sunset. (2%)

   (c) Explain why the cloud in the sky is WHITE. (2%)

10. (7%) Suppose that a monochromatic plane wave incidents on the boundary between two homogeneous, isotropic, and lossless dielectric media, with the indices n_i and n_t, as shown in the following figure.

   (a) Derive the Fresnel Equation for the amplitude reflection coefficient, $r_{//} \equiv \dfrac{E_r}{E_i}$, when **E**-field lines in the plane-of-incidence. (4%)

   (b) If the incident **E**-field is un-polarized, and at what angle of incidence one can make the reflected E-field become polarized. (3%)