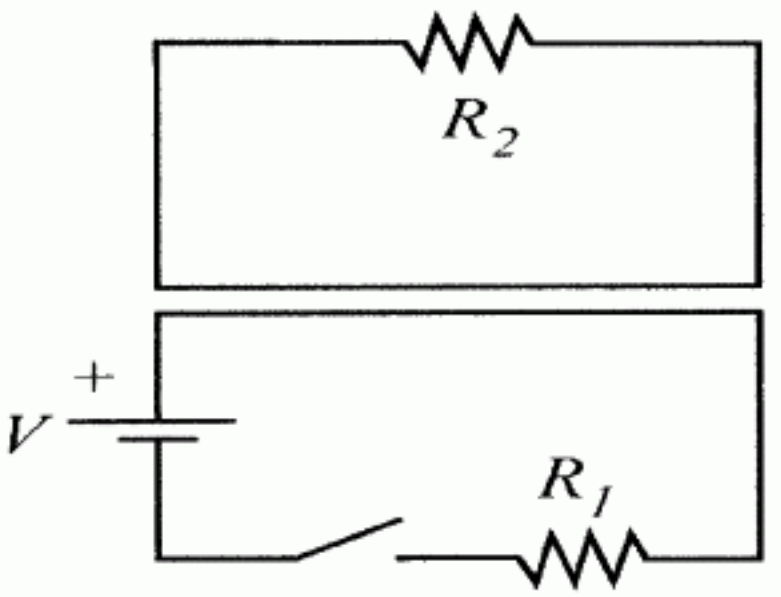
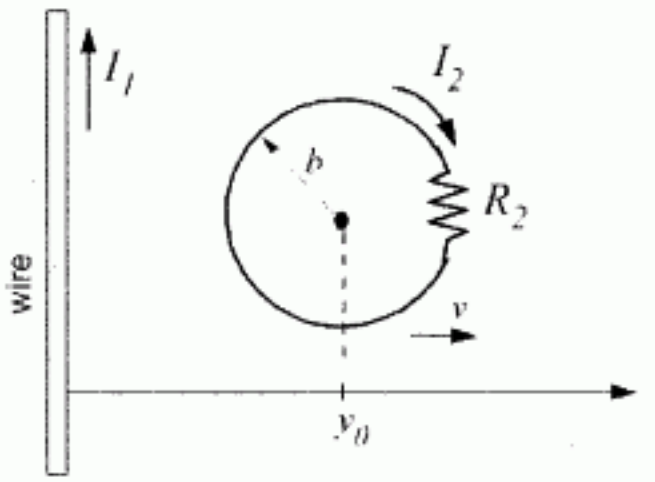
1. (15%) A sphere of radius R carries a non-uniform polarization, $\mathbf{P} = \mathbf{a}_r\, kr$, $0 < r < R$, where k is a constant. (a) Calculate the equivalent bound charges. (b) Find the **E** field inside and outside the sphere. Consult the last page for the vector calculus in the cylindrical and spherical coordinate systems.
2. (15%) Two circular loops of radius R are stacked distance d apart with their axes coincident with the z axis (assuming the origin is located in the middle of these two loops). If all carry the same current I in the same direction, calculate the magnetic flux density **B** along the z axis. Find the position where the magnetic flux density is maximum and determine its magnitude? Consult the last page for the vector calculus in the cylindrical and spherical coordinate systems.
3. (8%) The switch in the circuit below is closed a t $t = 0$ and then opened at a later time t_1. What is the direction of the current in the top loop at each of these times (*i.e.* clockwise or counterclockwise)? (3%) (Make a *qualitative* sketch of the current in the top loop versus time that you would expect for this circuit. (5%)



4. (7%) The resistively-loaded circular loop shown below moves away from a wire carrying a current $I_1 = 1$ A at a constant velocity $v = 5$ m/s, passing the point $y_0 = 10$ cm at $t = 0$. If $R_2 = 10\ \Omega$, find I_2 as a function of time. (Note that the mutual inductance between the wire and the loop is $L_{21} = \mu N[y_0 - \sqrt{y_0^2 - b^2}]$)

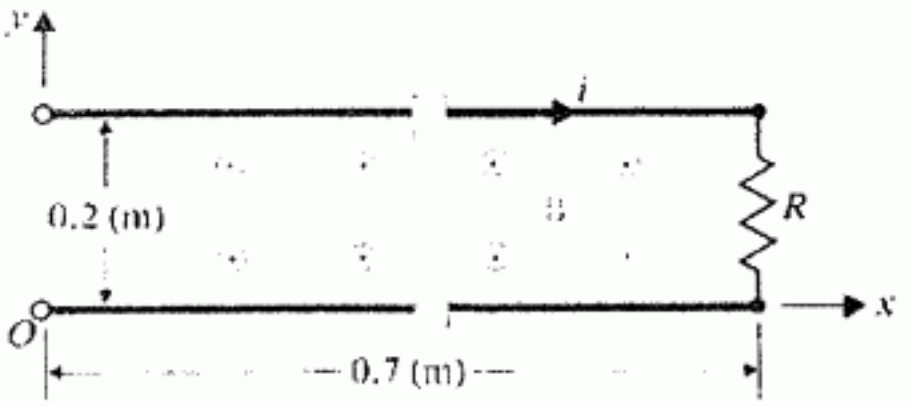


5. (9%) A microwave oven cooks food by irradiating the food with microwave power generated by a magnetron. The operating frequency is usually set at 2.45 GHz. For a beef steak that has approximately a dielectric constant of 40 and a loss tangent of 0.35 at 2.45 GHz, (a) find the effective conductivity of the lossy medium. (3%) (b) Estimate the average power dissipated per unit volume. (3%) (c) Considering skin effect, what's the impact on your answer (b) (3%)

6. (6%) A conducting sliding bar oscillates over two parallel conducting rails in a sinusoidally varying magnetic field

$$\mathbf{B} = \mathbf{a}_z\, 5 \cos \omega t \ (\text{mT})$$

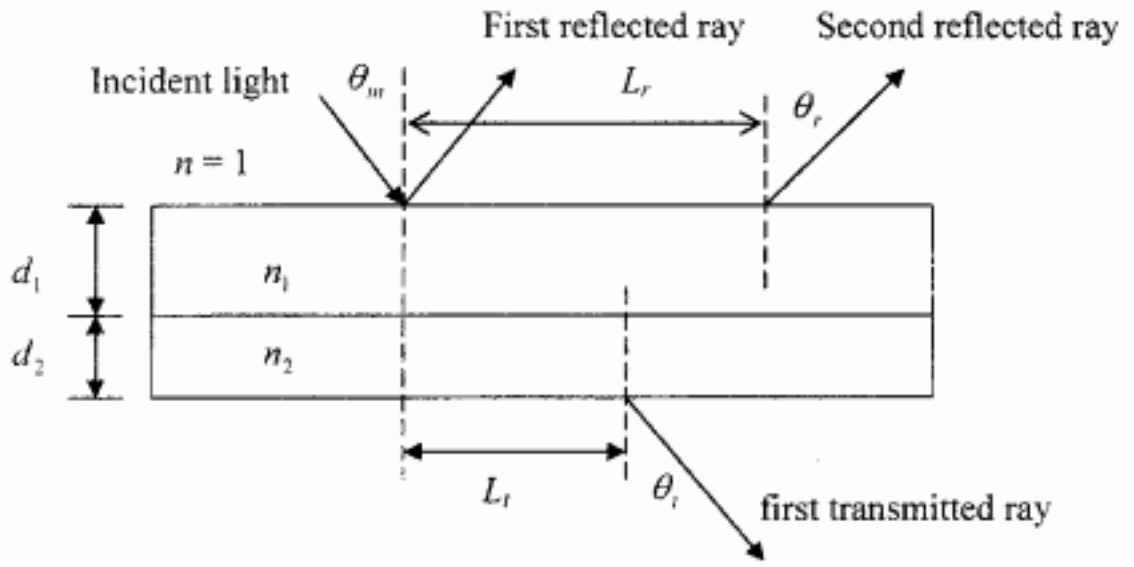
as shown in the figure below. The position of the sliding bar is given by $x = 0.35(1 - \cos \omega t)$ (m), and the rails are terminated in a resistance $R = 0.2\Omega$. Please find i.



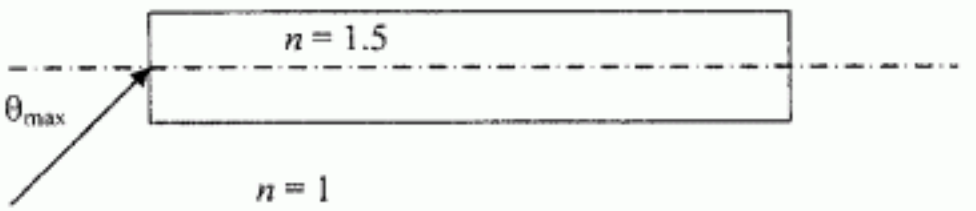
7. (24%) Answer the following questions.
   a. What is the approximate vacuum wavelength of a violet light? (3%)
   b. What is the approximate optical frequency of a red light? (3%)
   c. Lights of different frequencies see different refractive indices in a material. Among violet, green, yellow, and red lights, which one sees a larger refractive index in a typical glass material? (3%)
   d. A light has a wavelength of 500 nm in vacuum. What is the wavelength of this light in a material with a refractive index = 2? (3%)
   e. A monochromatic light is incident on an optically polished material surface and 10% of the incident electric field is reflected from the surface, what is the percentage of the incident intensity reflected from the surface? (3%)
   f. Given a photon frequency v, what is the energy of a photon? In your answer, define your symbols. (3%)
   g. Given a photon wave vector $\vec{k}$, what is the momentum of a photon? Define your symbols in your answer. (3%)
   h. Imagine that you are driving on a long straight highway and the sun straight ahead of you scatters a lot of lights into your eyes from the surface of the highway. What is the polarization direction of the most scattered lights relative to the plane of incidence? Give a scientific explanation to your answer. No explanation, no credit. (3%)

8. (6%) A light ray is incident on a transparent slab at an angle θ_{in} in vacuum, as shown below. The transparent slab consists of two dielectric layers with thickness d_1, d_2, and refractive indices n_1, n_2 for the first layer and second layer, respectively. Let $1 < n_1 < n_2$. Partial transmission and reflection occur at the vacuum-dielectric interfaces. Assume not reflection occurs at the junction of the first and second dielectric layers.
   a. What are the transmission angle θ_t and the lateral displacement L_t of the first transmitted ray? (3%)
   b. What are the reflection angle θ_r and the lateral displacement L_r of the second reflected ray? (3%)

In your calculation, assume a small incident angle θ_{in} and, for all angles in your calculation, let $\sin\theta \approx \tan\theta \approx \theta$.



9. (5%) A dielectric slab waveguide in vacuum has a refractive index of 1.5, as shown below. What is the maximum angle of an incident light within which the light can be guided in the waveguide?



10. (5%) A positive thin lens, made from a material of refractive index = 2.3, has a focal length of 10 cm in vacuum. What is its focal length, if this thin lens is submerged into water of refractive index = 1.3?

Vector Calculus for Problems 1 and 2.

Cylindrical coordinates

$$\nabla f = \hat{\rho}\frac{\partial f}{\partial r} + \hat{\phi}\frac{1}{r}\frac{\partial f}{\partial \phi} + \hat{z}\frac{\partial f}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r}\frac{\partial}{\partial r} rA_r + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \hat{\rho}\left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{\phi}\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \hat{z}\frac{1}{r}\left(\frac{\partial}{\partial r} rA_\phi - \frac{\partial A_r}{\partial \phi}\right) = \begin{vmatrix} \hat{\rho}\frac{1}{r} & \hat{\phi} & \hat{z}\frac{1}{r} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix}$$

Spherical coordinates

$$\nabla f = \hat{\rho}\frac{\partial f}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial f}{\partial \theta} + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r} r^2 A_r + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \mathbf{A} = \hat{\rho}\frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(A_\phi \sin\theta) - \frac{\partial A_\theta}{\partial \phi}\right] + \hat{\theta}\frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{\partial}{\partial r} rA_\phi\right) + \hat{\phi}\frac{1}{r}\left(\frac{\partial}{\partial} rA_\theta - \frac{\partial A_r}{\partial \theta}\right)$$