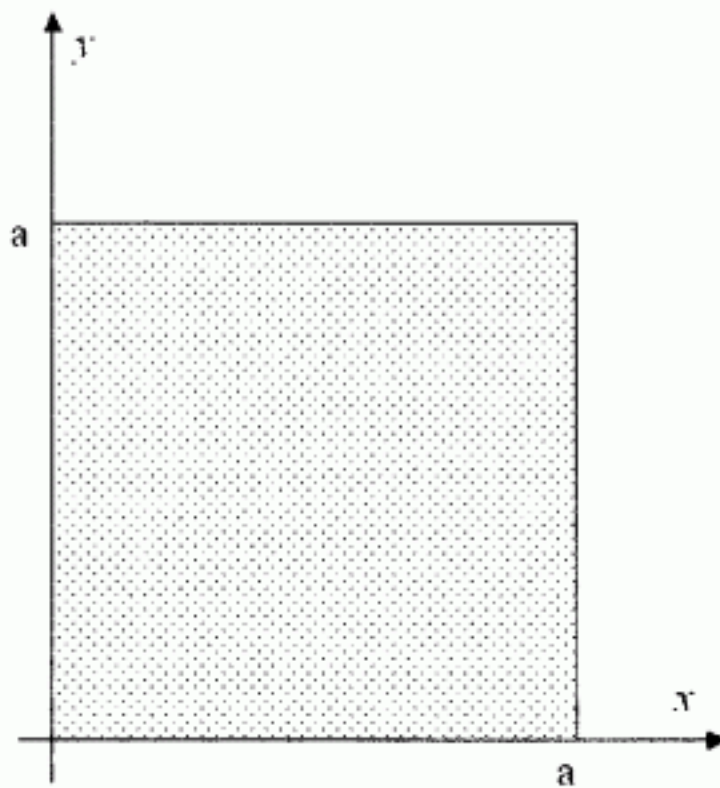


國立清華大學命題紙

九十三學年度 光電工程研究所 系(所) _____ 組碩士班入學考試

科目 工程數學 科號 2801 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

1. (20%) Find the steady-state temperature $u(x,y)$, where $\nabla^2 u(x,y) = 0$, in a square plate ($a \times a$) with upper and lower sides perfectly insulated, the left side kept at 0°C , and the right side at $f(y)^\circ\text{C}$, where $f(y) = \frac{a}{2} - |y - \frac{a}{2}|$.



2. (10%) Find the integral $\int_{-\infty}^{\infty} \cos(2\pi\alpha) \left[\frac{\sin[2\pi(x-\alpha)]}{(x-\alpha)} \right]^2 d\alpha$.

3. (10%) Find $L^{-1}[e^{-2t}f(t)]$, where

$$f(t) = \begin{cases} 0, & t < 4 \\ t^2 - 4, & t \geq 4 \end{cases}$$

4. (15%) Solve the equation $x^2 y'' - (x + x^2) y' + y = 0$ ($0 < x < \infty$)

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5. (20%) Evaluate

$$\int_{-\infty}^{\infty} \frac{\cos(\alpha x)}{(x^2 + \alpha^2)(x^2 + \beta^2)} dx,$$

where σ , α , and β are positive constants and $\alpha \neq \beta$.

6. (10%) Let $F(x,y) = |x + y|$ and $G(x,y) = |x - y|$. Compute the integral

$$\iint_R (\partial F / \partial x - \partial G / \partial y) dx dy$$

over the area bounded by a circle having a radius of unity (i.e., $r = 1$) and centered at the origin of x - y plane. (Note that derivation of your answer is a must. Without derivation, a correct answer would deserve no point. Nevertheless, some points might be given to your derivation even your answer is wrong.)

7. (15%) If an $n \times n$ matrix A has fewer than n linearly independent eigenvectors, we say that A is *defective*. For each of the following matrices, find all possible values of the scalar α that make the matrix defective or show that no such values exist.

(a)
$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & \alpha \end{bmatrix}$$

(b)
$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 2 & -1 & \alpha \end{bmatrix}$$

(c)
$$\begin{bmatrix} 4 & 6 & -2 \\ -1 & -1 & 1 \\ 0 & 0 & \alpha \end{bmatrix}$$