國 立 清 華 大 學 命 題 紙

九十二學年度 光電工程研究 系（所） 組碩士班研究生招生考試

科目 工程數學 科號 2501 共 二 頁第 一 頁 *請在試卷【答案卷】內作答

1. Solve $y''+2y'+y=0$ for the general solution y(x). (5%)

2. Derive the general solution y(x) for the equation
$y'+f(x)y=g(x)$. (10%)

3. Write a partial differential equation that has the solution
$y(x,t)=(4x+8)\cdot(-t/4+1)$. Note that $\partial y/\partial x$ and $\partial y/\partial t$ must be included in the equation. Also note that you have to give a reason for bringing up your answer, i.e., you have to show how you derive your answer. (15%)

4. The velocity of a rotating particle is given by a vector $\mathbf{V}=\boldsymbol{\omega}\times\mathbf{r}$, where $\boldsymbol{\omega}$ is a constant vector. Find the value of $\nabla\times\mathbf{V}$. (5%)

5. Find the surface integral of the vector function $\mathbf{F}=x\,\mathbf{i}+y\,\mathbf{j}+z\,\mathbf{k}$ over that portion of the surface $z=xy+1$, which covers the square $0\le x\le 1$, $0\le y\le 1$ in the xy plane. (10%)

6. Find all Taylor or Laurent series representations with center $z_0=1$, and their corresponding precise region of convergence of the function $f(z)=\sinh z/(z-1)^2$. (10%)

7. Decide whether the following matrices are positive definite, negative definite, or indefinite? (a) $\begin{pmatrix}3 & \sqrt{2}\\ \sqrt{2} & 4\end{pmatrix}$ (b) $\begin{pmatrix}-2 & 0 & 1\\ 0 & -1 & 0\\ 1 & 0 & -2\end{pmatrix}$ (c) $\begin{pmatrix}6 & 4 & -2\\ 4 & 5 & 3\\ -2 & 3 & 6\end{pmatrix}$ (15%)

8. Assume that the coefficient of complex Fourier series of a periodic function $f_1(t)$ with period = 4 shown in Fig. 1 is c_n, where $-\infty < n < \infty$.

(a) Find the complex Fourier series representation for a periodic function $f_2(t)$ with the same period (= 4) shown in Fig. 2 in terms of c_n. (8%)

(b) Find the Fourier series representation of the solution y(t) in terms of c_n if

$$\frac{d^2y(t)}{dt^2} + \frac{dy(t)}{dt} + y(t) = f_2(t), \quad -\infty < t < \infty. \quad (7\%)$$

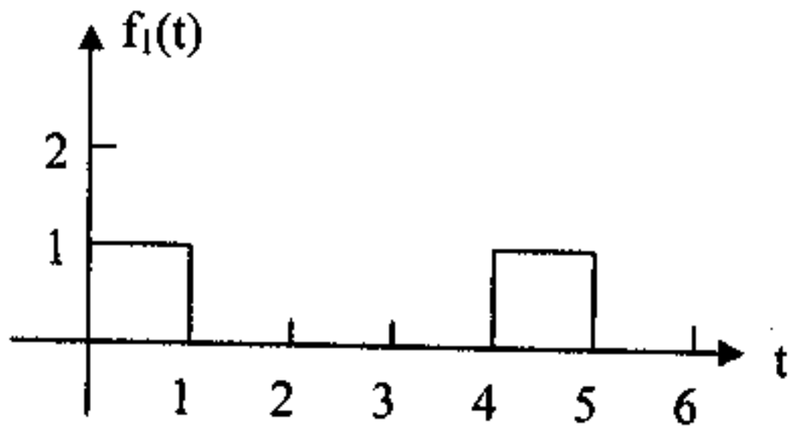

Fig. 1

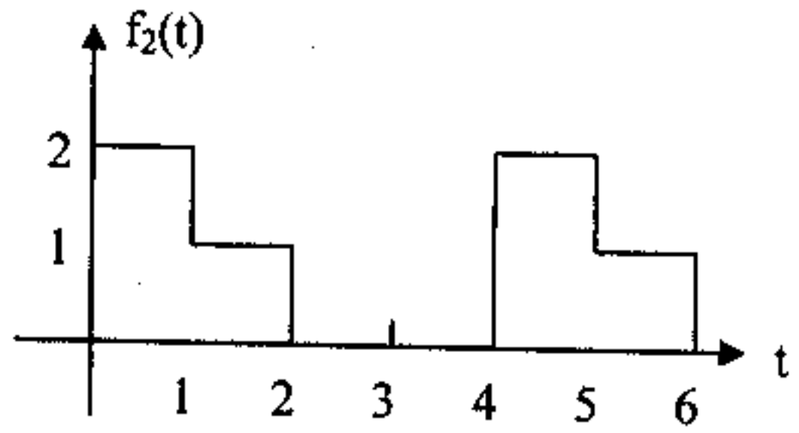

Fig. 2

9. The capacitor voltage $v_o(t)$ in a series RLC circuit satisfies the following differential equation:

$$\frac{1}{2}\frac{d^2v_o(t)}{dt^2} + \frac{3}{2}\frac{dv_o(t)}{dt} + v_o(t) = v_i(t), \qquad \left.\frac{dv_o(t)}{dt}\right|_{t=0^+} = 2, v_o(0^+) = 1.$$

Use Laplace transform to find the capacitor voltage $v_o(t)$ for $t > 0$ if the voltage source $v_i(t) = e^{-3t}u(t)$. (15%)