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※請在答案卷內作答

Note: Detailed Derivations are required to obtain a full score for each problem.

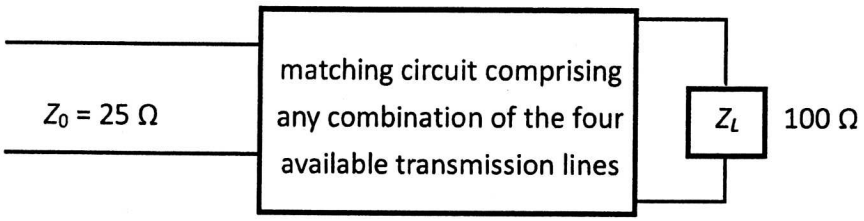
1. (10 pts) What is the positon vector **R** to an arbitrary point (x, y, z) in Cartesian coordinates? And what is the divergence of **R**? Here the base vectors are $\mathbf{a}_x$, $\mathbf{a}_y$, and $\mathbf{a}_z$.

2. (5 pts) When the dielectric medium is lossy (having a small but non-zero conductivity σ), the capacitance between two conductors separated by this dielectric medium can be defined as C while the resistance between these two conductors is R. Please prove that $RC = \epsilon/\sigma$. Here ϵ is the permittivity of the medium.

3. Consider oblique incidence from medium 1 to medium 2 problems:
   (a) (10 pts) Derive the reflection coefficient and transmission coefficient with perpendicular polarization incidence at a plane dielectric boundary.
   (b) (10 pts) Derive the reflection coefficient and transmission coefficient with parallel polarization incidence at a plane dielectric boundary.
   (c) (10 pts) Define the Brewster angle and the critical angle, respectively.

參考用

4. (10 pts) A load Z_L = 100 Ω is to be matched to a transmission line with characteristic impedance Z_0 = 25 Ω. Four quarter-wave transmission lines, each of length = $\lambda/4$, with various characteristic impedances: Z_1 = 45 Ω , Z_2 = 60 Ω , Z_3 = 75 Ω, and Z_4 = 90 Ω are available. Find a way to achieve this matching by using any combination of these four available transmission lines and draw the matching circuit. Explicit working must be shown to motivate or explain the solution. Hint: Not necessary to use all four transmission lines.



5. The standing-wave ratio on a lossless 50Ω transmission line terminated in an unknown load impedance is found to be 3.0. The distance between successive voltage minima is 20 cm, and the first minimum is located at 5 cm from the load. Determine
   (a) (4 pts) the reflection coefficient,
   (b) (3 pts) the load impedance, and
   (c) (3 pts) the shortest possible equivalent length and the associated terminating resistance of such a transmission line inserted in place of the original load such that the input impedance seen into this replacement line towards its terminal resistance is equal to the original load impedance found in **(b)**.

注意：背面有試題

6. A dielectric-filled metallic waveguide of unknown permittivity has dimension a=2 cm and b=4 cm along x and y directions respectively. Now the magnetic field of a guided mode propagating along this metallic waveguide is known as

$$H = \hat{y}30\cos(100\pi x)\sin(200\pi y)\sin\left(2\pi\times10^{10}t - 557\pi z\right) \text{ (A/m)}$$

Determine

(a) (5 pts) the mode number of this guided mode

(b) (5 pts) ε_r (dielectric constant) of the filled dielectric material

(c) (5 pts) the cutoff frequency of this guided mode

7. Consider a InGaAsP-InP laser diode which has an optical cavity of length 150 microns. The peak radiation is at 1550 nm and the refractive index of InGaAsP is 4. The optical gain bandwidth (as measured between half intensity points) will normally depend on the pumping current (diode current) but for this problem assume that it is 5 nm.

(a) (5 pts) What is the mode integer m of the peak radiation?

(b) (5 pts) What is the separation between the modes of the cavity? Please express your answer as $\Delta\lambda$.

(c) (5 pts) How many modes are within the gain band of the laser?

(d) (5 pts) What is the reflection coefficient and reflectance at the ends of the optical cavity (faces of the InGaAsP crystal)?