共 4 頁 第 1 頁

※請在答案卡內作答

- 本測驗試題為複選題(答案可能有一個或多個),請選出所有正確或最適當的答案,並請用2B鉛筆作答於答案卡。
- 共二十題,每題完全答對得五分,答錯不倒扣。

Notation: In the following questions, underlined letters such as $\underline{a}, \underline{b}$, etc. denote column vectors of proper length; boldface letters such as \mathbf{A}, \mathbf{B} , etc. denote matrices of proper size; \mathbf{A}^{\top} means the transpose of matrix \mathbf{A} . \mathbb{R} is the usual set of all real numbers. By $\mathbf{A} \in \mathbb{R}^{m \times n}$ we mean \mathbf{A} is an $m \times n$ real-valued matrix. u(x) is unit-step function defined as u(x) = 1 if $x \geq 0$ and u(x) = 0 if x < 0; \star is the convolution operator; $\mathcal{L}: f(x) \mapsto F(s)$ and $\mathcal{L}^{-1}: F(s) \mapsto f(x)$ denote the <u>unilateral</u> Laplace and inverse Laplace transforms for $x \geq 0$, respectively.

- • To solve a system of linear equations, we often use the augmented matrix form $[A \ \underline{b}]$; then reduce A to an upper triangular matrix by operating on the rows of A and carry out the same operations on the vector \underline{b} . For instance, consider

$$\begin{cases} x + 2y + 2z &= 1\\ 4x + 8y + 9z &= 3\\ 3y + 2z &= 1 \end{cases}$$
 (1)

so we have

$$\begin{bmatrix} \mathbf{A} & \underline{b} \end{bmatrix} = \begin{bmatrix} 1 & 2 & 2 & 1 \\ 4 & 8 & 9 & 3 \\ 0 & 3 & 2 & 1 \end{bmatrix}.$$

Then we can apply an elimination matrix \mathbf{E} to the system such that $\mathbf{E}[\mathbf{A}\ \underline{b}] = [\mathbf{U}\ \underline{c}]$ and \mathbf{U} is an upper triangular matrix. Which of the following statements are true?

- (A) Left-multiply matrix \mathbf{A} by $\mathbf{E}_{21} = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ to subtract four times the first row of \mathbf{A} from the second row of \mathbf{A} .
- (B) Left-multiply matrix $\bf A$ by $\bf P_{32}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ to exchange the second and the third columns of $\bf A$.
- (C) We can do both steps at once, i.e., set $\mathbf{E} = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ and $\mathbf{U} = \mathbf{E}\mathbf{A}$ is an upper triangular matrix.
- (D) The solution to the linear system (1) is x = 1, y = 1 and z = -1.
- (E) None of the above is true.

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類組: <u>電機類</u> 科目: 工程數學 C(3005)

共14頁第2頁

※請在答案卡內作答

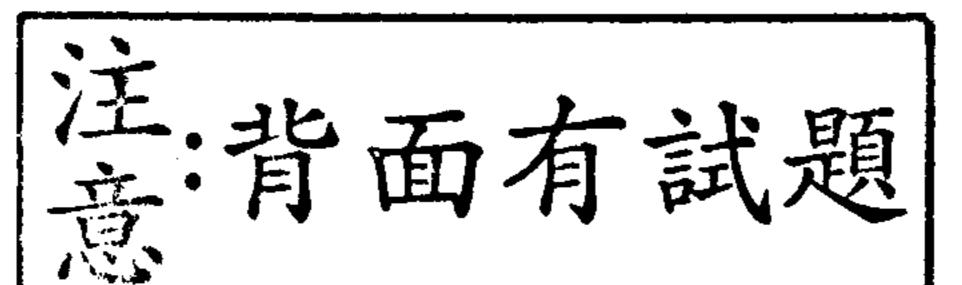
= Suppose that matrix A can be factorized into the following form

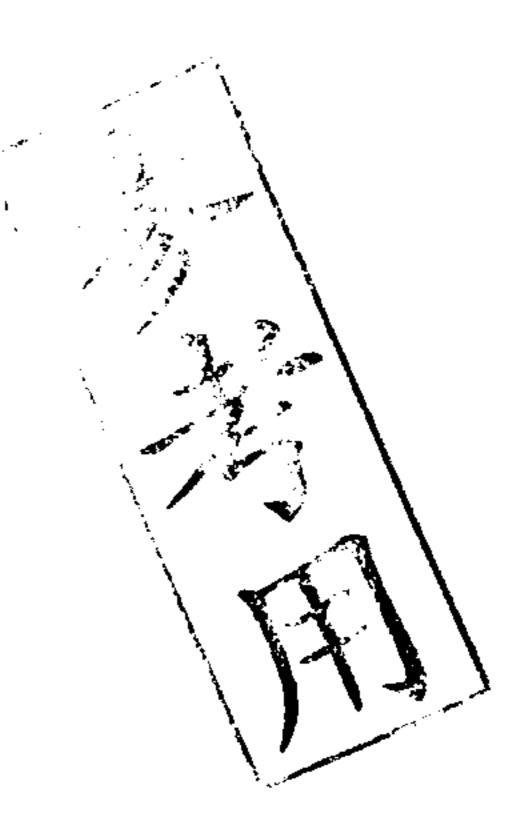
$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 0 & 5 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Which of the following statements are true?

- (A) $rank(\mathbf{A}) = 2$
- (B) The dimension of the row-space of A equals 3.
- (C) The dimension of the column-space of A equals 2.
- (D) The dimension of the nullspace of A equals 1.
- (E) None of the above is true.

- Ξ Let $A, B \in \mathbb{R}^{4 \times 4}$ be matrices. Suppose that A and B have the same column space, but may not have the same columns. Which of the following statements are true?
 - (A) Both matrices A and B have the same number of pivots.
 - (B) Both matrices A and B have the same left nullspace and the same nullspace.
 - (C) If A is invertible, so is B.
 - (D) The row spaces of matrices **A** and **B** are orthogonal to each other under the usual Euclidean inner product.
 - (E) None of the above is true.





類組: <u>電機類</u> 科目: 工程數學 C(3005)

共14 頁第3 頁

※請在答案卡內作答

ष्य • Consider the matrix equation $\mathbf{A}\underline{x} = \underline{b}$, where

$$\mathbf{A} = \left[egin{array}{cccc} 1 & 1 \ 1 & -1 \ 2 & 0 \end{array}
ight] \quad ext{ and } \underline{b} = \left[egin{array}{cccc} 2 \ 0 \ 1 \end{array}
ight]$$

Let P be an orthogonal projection matrix from \mathbb{R}^3 onto the column space of A. Which of the following statements are true?

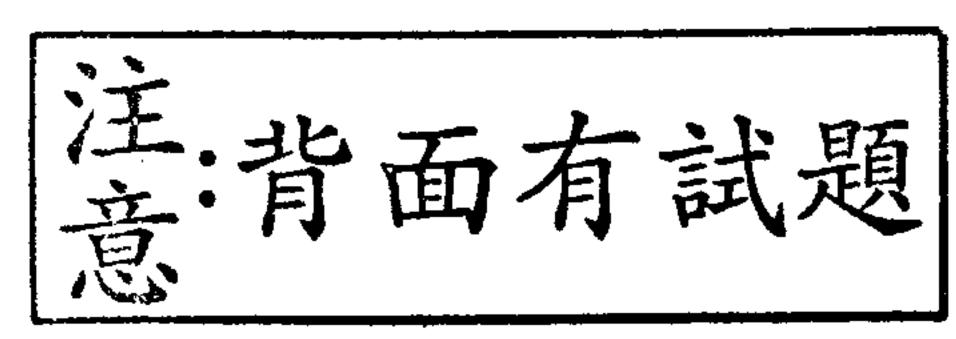
- (A) The matrix A is rectangular. It has no multiplicative left-inverse.
- (B) The solution to $\mathbf{A}\underline{\hat{x}} = \mathbf{P}\underline{b}$ is $\underline{\hat{x}} = \begin{bmatrix} 1 & 1 \end{bmatrix}^{\mathsf{T}}$.
- (C) The projection vector $\mathbf{P}\underline{b} = \frac{1}{3}[2 \ 3]^{\mathsf{T}}$.
- (D) $\mathbf{P} = \mathbf{A} (\mathbf{A}^{\mathsf{T}} \mathbf{A})^{-1} \mathbf{A}^{\mathsf{T}}$; \mathbf{P} is symmetric and satisfies $\mathbf{P}^2 = \mathbf{P}$.
- (E) None of the above is true.

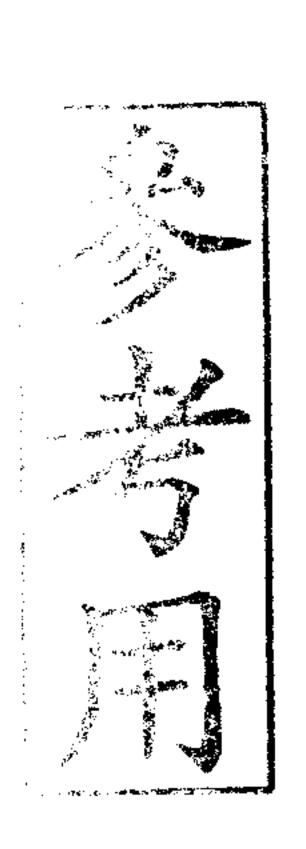


$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 3 \\ 1 & 3 & 1 & 9 \end{bmatrix}$$

which of the following statements are true?

- $(A) \det(\mathbf{A}) = 3$
- (B) A is invertible
- $(C) \det(\mathbf{A}^{-1}) = 3$
- (D) The (1,2)th entry of A^{-1} equals 3.
- (E) None of the above is true.





類組: <u>電機類</u> 科目: 工程數學 C(3005)

共14 頁第4 頁

※請在答案卡內作答

 $\dot{\pi}$ What are the coordinate vectors for $f(x) = 5x^2 + 2x - 3$, with ordered bases $\mathcal{B}_1 = \{x^2, x, 1\}$ and $\mathcal{B}_2 = \{x^2 - x + 1, 3x^2 + 1, 2x^2 + x - 2\}$, respectively?

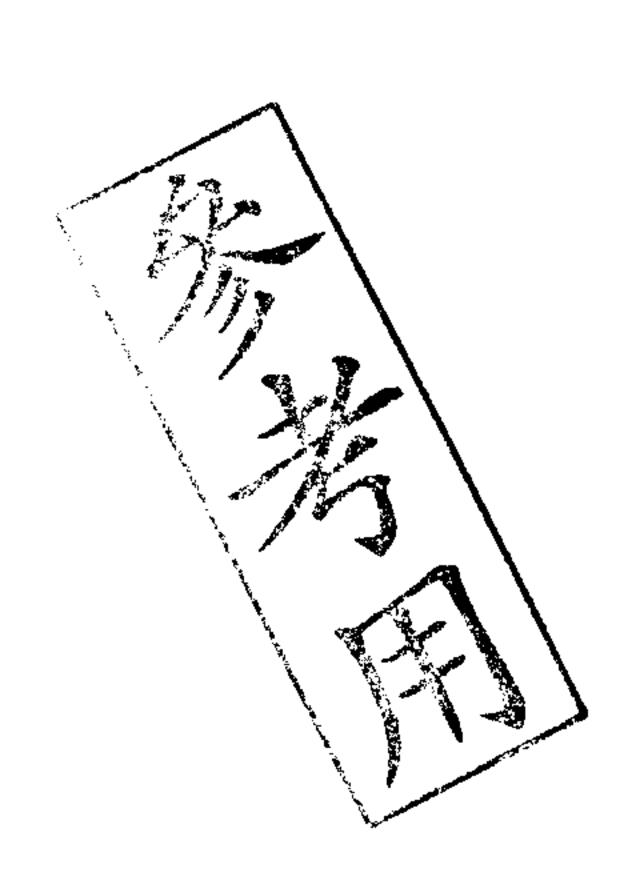
(A)
$$[f(x)]_{\mathcal{B}_1} = \begin{bmatrix} -3 \\ 2 \\ 5 \end{bmatrix}$$
, $[f(x)]_{\mathcal{B}_2} = \begin{bmatrix} 0 \\ -\frac{13}{3} \\ \frac{5}{3} \end{bmatrix}$.

(B)
$$[f(x)]_{\mathcal{B}_1} = \begin{bmatrix} 0 \\ -\frac{13}{3} \\ \frac{5}{3} \end{bmatrix}$$
, $[f(x)]_{\mathcal{B}_2} = \begin{bmatrix} 5 \\ 2 \\ -3 \end{bmatrix}$

(C)
$$[f(x)]_{\mathcal{B}_1} = \begin{bmatrix} \frac{2}{3} \\ -\frac{13}{3} \\ 0 \end{bmatrix}, [f(x)]_{\mathcal{B}_2} = \begin{bmatrix} 5 \\ 2 \\ -3 \end{bmatrix}.$$

(D)
$$[f(x)]_{\mathcal{B}_1} = \begin{bmatrix} \frac{2}{3} \\ -\frac{13}{3} \\ 0 \end{bmatrix}, [f(x)]_{\mathcal{B}_2} = \begin{bmatrix} 2 \\ 5 \\ -3 \end{bmatrix}.$$

(E) None of the above is true.



七 \ Given a non-zero matrix $\mathbf{A} \in \mathbb{R}^{n \times n}$, which of the following statements are true?

- (A) The nullspace of A equals the left nullspace of A.
- (B) The eigenvectors of A are linearly independent.
- (C) Part of the eigen-space can equal the left nullspace of A.
- (D) The dimension of the eigen-space of A equals n.
- (E) None of the above is true.

共14 頁第5

※請在答案卡內作答

 \wedge Let T be a linear operator on \mathbb{R}^3 defined as

$$T(\underline{x}) = \left[\begin{array}{c} 2x_2 + x_3 \\ x_1 - 4x_2 \\ 3x_1 \end{array} \right], \text{ with } \underline{x} = \left[\begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} \right].$$

What is the standard matrix, $[T]_{\mathcal{B}}$, associated to T with respect to the ordered basis

$$\mathcal{B} = \left\{ egin{array}{c} \underline{v}_1 = \left[egin{array}{c} 1 \\ 1 \\ 1 \end{array}
ight], \underline{v}_2 = \left[egin{array}{c} 1 \\ 1 \\ 0 \end{array}
ight], \underline{v}_3 = \left[egin{array}{c} 1 \\ 0 \\ 0 \end{array}
ight]
ight\},$$

and $[T(\underline{x})]_{\mathcal{B}}$?

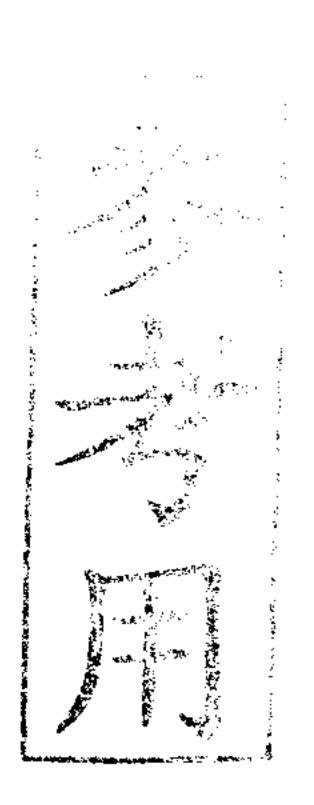
$$(A) \ [T]_{\mathcal{B}} = \begin{bmatrix} 0 & 2 & 3 \\ 1 & -4 & 0 \\ 3 & 0 & 0 \end{bmatrix}, \ [T(\underline{x})]_{\mathcal{B}} = \begin{bmatrix} x_3 \\ x_2 - x_3 \\ x_1 - x_2 \end{bmatrix}.$$

(B)
$$[T]_{\mathcal{B}} = \begin{bmatrix} 3 & 3 & 3 \\ -6 & -6 & -2 \\ 6 & 5 & -1 \end{bmatrix}, [T(\underline{x})]_{\mathcal{B}} = \begin{bmatrix} 3x_1 \\ -2x_1 - 4x_2 \\ -x_1 + 6x_2 + x_3 \end{bmatrix}.$$

(C)
$$[T]_{\mathcal{B}} = \begin{bmatrix} 3 & 3 & 3 \\ -6 & -6 & -2 \\ 6 & 5 & -1 \end{bmatrix}, [T(\underline{x})]_{\mathcal{B}} = \begin{bmatrix} x_3 \\ x_2 - x_3 \\ x_1 - x_2 \end{bmatrix}$$

(D)
$$[T]_{\mathcal{B}} = \begin{bmatrix} 0 & 2 & 3 \\ 1 & -4 & 0 \\ 3 & 0 & 0 \end{bmatrix}$$
, $[T(\underline{x})]_{\mathcal{B}} = \begin{bmatrix} 3x_1 \\ -2x_1 - 4x_2 \\ -x_1 + 6x_2 + x_3 \end{bmatrix}$.

(E) None of the above is true.



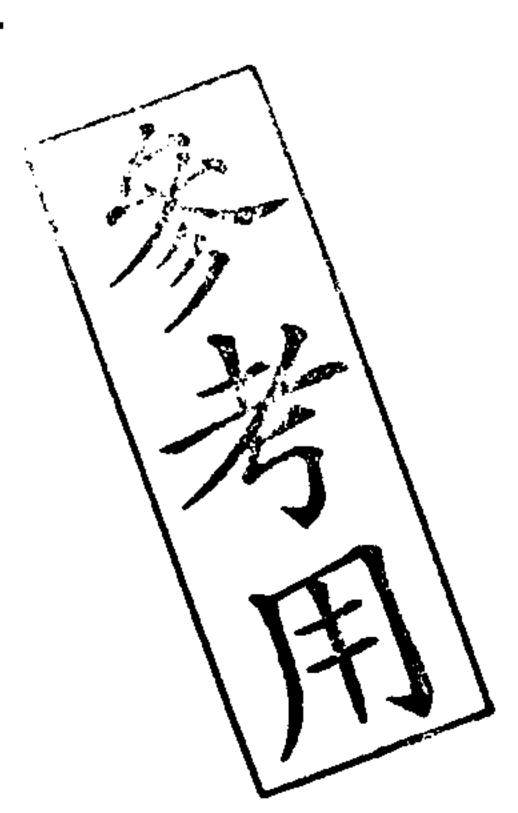
類組: <u>電機類</u> 科目: <u>工程數學 C(3005)</u>

共14頁第6頁

※請在答案卡內作答

- た、Let $\mathbf{A} \in \mathbb{R}^{\mathbf{n} \times \mathbf{n}}$ be a non-zero, symmetric matrix. Suppose the eigen-decomposition of $\mathbf{A} = \mathbf{Q} \mathbf{\Lambda} \mathbf{Q}^{\top}$, where $\mathbf{\Lambda}$ is a diagonal matrix and $\mathbf{Q} = \begin{bmatrix} \mathbf{Q}_1 & \mathbf{Q}_2 \end{bmatrix}$ is an orthonormal matrix, with $\mathbf{Q}_1 \in \mathbb{R}^{n \times r}$ and $\mathbf{Q}_2 \in \mathbb{R}^{n \times (n-r)}$, $r = \operatorname{rank}(\mathbf{A})$. Which of the following statements are true?
 - (A) span $(\underline{x} \mathbf{Q}_1 \mathbf{Q}_1^{\mathsf{T}} \underline{x}) = \operatorname{span} (\mathbf{Q}_2 \mathbf{Q}_2^{\mathsf{T}} \underline{x})$ for any given vector $\underline{x} \in \mathbb{R}^n$.
 - (B) \mathbf{AQ}_2 is <u>not</u> an all-zero matrix.
 - (C) $\mathbf{Q}_1^{\mathsf{T}}\mathbf{Q}_2$ is an all-zero matrix.
 - (D) $\mathbf{Q}_2^{\mathsf{T}}\mathbf{Q}_2$ is an all-zero matrix.
 - (E) None of the above is true.

- 十、 Which of the following statements are true?
 - (A) A real matrix may be positive definite without being symmetric with respect to the real Euclidean inner product.
 - (B) A positive semidefinite matrix cannot have 0 as an eigenvalue.
 - (C) det: $\mathbb{R}^{n \times n} \to \mathbb{R}$ is a linear transformation.
 - (D) Let **A** and **B** be square matrices; then $det(\mathbf{A}^{\mathsf{T}}\mathbf{B}) = det(\mathbf{A}) det(\mathbf{B})$.
 - (E) None of the above is true.



類組: <u>電機類</u> 科目: 工程數學 C(3005)

共14 頁第一丁頁

※請在答案卡內作答

十一、 Consider the following non-homogeneous system:

$$\begin{cases} y_1'(x) = 4y_1(x) + 2y_2(x) - 15xe^{-2x} \\ y_2'(x) = 3y_1(x) - y_2(x) - 4xe^{-2x} \end{cases}$$

satisfying $y_1(0) = 7$ and $y_2(0) = 3$. Which of the following statements are true?

(A)
$$y_1(x) = (6 + 2x - 7x^2)e^{-2x} + e^{5x}$$
 and $y_2(x) = (-4 + 21x^2)e^{-2x} + 7e^{5x}$.

(B)
$$y_1(x) = \frac{14+13x^2}{14}e^{-2x} + \frac{42+4x}{7}e^{5x}$$
 and $y_2(x) = \frac{28+5x}{14}e^{-2x} + \frac{7+10x+6x^2}{7}e^{5x}$.

(C)
$$y_1(1) = \frac{27}{14}e^{-2} + \frac{46}{7}e^5$$
 and $y_2(1) = \frac{31}{14}e^{-2} + \frac{23}{7}e^5$.

(D)
$$y_1(-1) = -3e^2 + e^{-5}$$
 and $y_2(-1) = 17e^2 + 7e^{-5}$.

(E) None of the above is true.

$$+$$
: Consider the differential equation $(2xy(x) - x^2 - y^2(x) - 1)dx + dy(x) = 0$. Which of the following statements are true?

- (A) The differential equation is exact.
- (B) x + y(x) = 0 is an implicit solution.

(C) If
$$y(0) = \frac{1}{2}$$
, then $y(1) = 2$.

(D) If
$$y(0) = 1$$
, then $y(2) = 1$.



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類組: <u>電機類</u> 科目: 工程數學 C(3005)

共14 頁第 8 頁

※請在答案卡內作答

 $+ \equiv$ Consider the differential equation $xdy(x) - 2y(x)[2x^2 - \ln y(x)]dx = 0$. Which of the following statements are true?

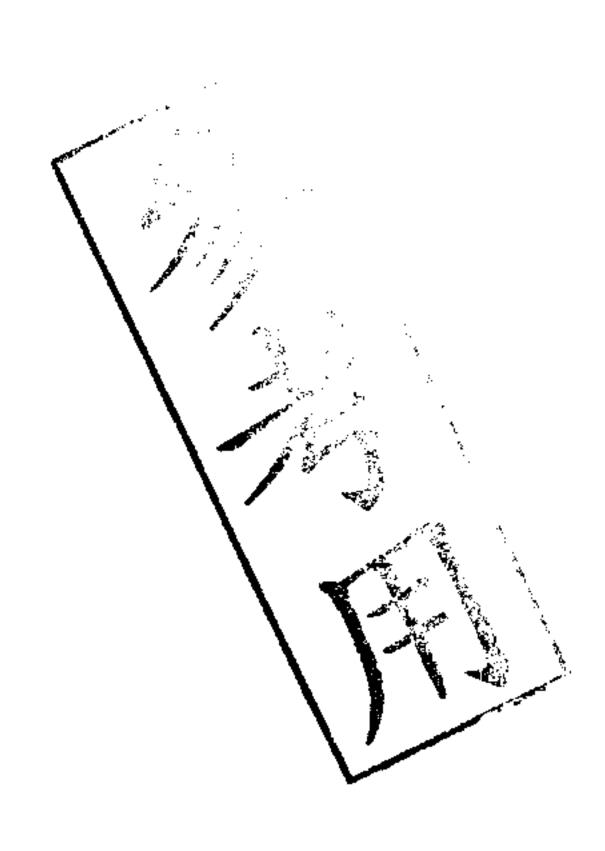
(A) The differential equation is linear.

(B) $y(x) = e^{x^2}$ is a particular solution.

(C) If y(1) = 1, then $y(2) = \frac{15}{4}$.

(D) If y(1) = 1, then y'(1) = 4.

(E) None of the above is true.



十四、 Consider the differential equation $(1-x^2)y''(x)-2xy'(x)+2y(x)=0$. Which of the following statements are true?

(A) $y(x) = \ln \frac{1+x}{1-x}$ is a particular solution.

(B) y(x) = x is a particular solution.

(C) If y(0) = 1 and y'(0) = 1, then $y(\frac{1}{2}) = \frac{3}{2} - \frac{\ln 3}{4}$.

(D) If y(0) = 2 and y'(0) = 0, then $y(\frac{1}{2}) = 2 + \ln 3$.

(E) None of the above is true.

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共 14 頁 第 9 頁

※請在答案卡內作答

十五、 Consider the following differential equation:

$$(x^3+1)y^{(3)}(x) + (3-x^2)y''(x) + (4+2x)y'(x) + 10y(x) = 4+12x + (4x-8x^3)\cos(2x) + (4x^2-2)\sin(2x)$$

satisfying $y(\pi) = \pi$, $y'(\pi) = 3$ and $y''(\pi) = 0$. Which of the following statements are true?

- (A) y(0) = 0.
- (B) $y(\frac{\pi}{2}) = \frac{\pi}{2}$.
- (C) $y'(\frac{\pi}{4}) = 1$.
- (D) $y'(1) = \pi$.
- (E) None of the above are true.



類組:<u>電機類</u>科目:工程數學 C(3005) ※請在答案卡內作答

共 14 頁 第 10 頁

 $+ \div$ Consider the system of differential equations $\underline{y}'(x) = \mathbf{A}\underline{y}(x) + \underline{f}(x)$, where

$$\mathbf{A} = \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}, \ \underline{y}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \ \mathrm{and} \ \underline{f}(x) = \begin{bmatrix} x^2 \\ x \end{bmatrix}.$$

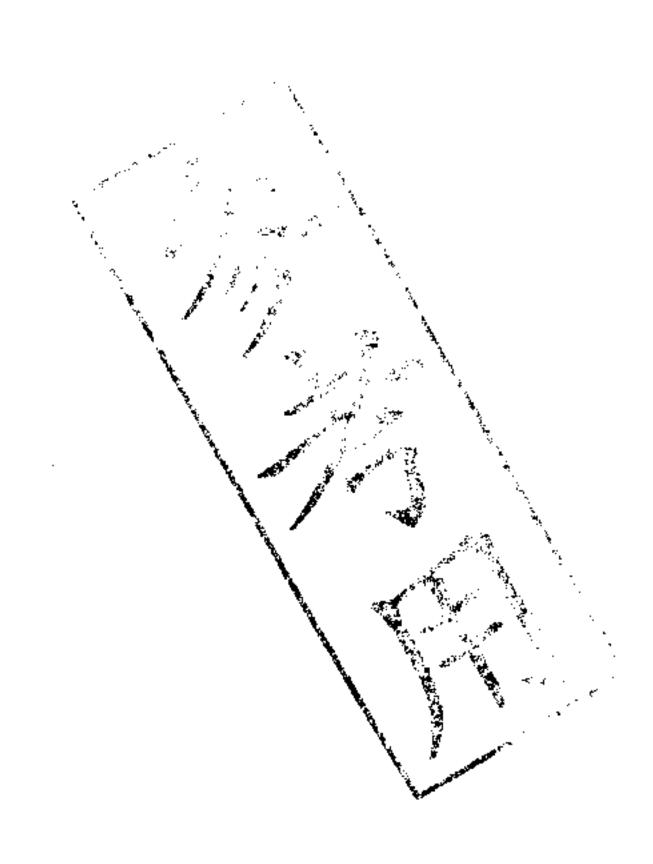
Which of the following statements are true?

(A)
$$e^{\mathbf{A}x}\underline{y}(0) = \begin{bmatrix} 1-3x\\1-x \end{bmatrix}$$

(B)
$$\mathscr{L}\{\underline{y}(x)\} = \frac{1}{s^5} \begin{bmatrix} s^4 - 2s^3 + 2s + 4 \\ s^4 - s^3 + s^2 - 2s + 2 \end{bmatrix}$$

(C)
$$\underline{y}(1) = \begin{bmatrix} -\frac{7}{6} & \frac{1}{4} \end{bmatrix}^{\mathsf{T}}$$

- (D) $[2\ 1]^{\top}$ is a generalized eigenvector for matrix **A** based on eigenvector $[1\ 0]^{\top}$.
- (E) None of the above is true.



十七、 Let g(x) be a real-valued function given as below

$$g(x) = \begin{cases} e^{-|x|}, & \text{if } x \in (-\pi, \pi), \\ 0, & \text{otherwise} \end{cases}$$

which has a Fourier series (F.S.) representation for $x \in (-\pi, \pi)$ as follows

$$g(x) \stackrel{\text{F.S.}}{=} g_0 + \sum_{n \ge 1} \left[a_n \cos(nx) + b_n \sin(nx) \right]$$

Which of the following statements are true?

共14頁第11頁

※請在答案卡內作答

(A)
$$g_0 = \frac{1 - e^{-\pi}}{\pi}$$

(B)
$$a_1 = \frac{1 - e^{-\pi}}{\pi}$$

(C)
$$b_1 = 0$$

(D)
$$2g_0^2 + \sum_{n\geq 1} (a_n^2 + b_n^2) = \frac{1-e^{-2\pi}}{\pi}$$

(E) None of the above is true.



十八、 Continued from Question 十七, solve for y(x) the following differential equation

$$y''(x) - 9y(x) = \sum_{n=-\infty}^{\infty} g(x - 2n\pi).$$

for all $x \in \mathbb{R}$ and $x \neq \pm \pi, \pm 3\pi, \pm 5\pi, \ldots$ With y(0) = y'(0) = 0, it is already known that the solution y(x) takes the following form

$$y(x) \stackrel{\text{F.S.}}{=} y_0 + \sum_{n\geq 1} \left[c_n \cos(nx) + d_n \sin(nx) \right]$$

Which of the following statements are true?

(A)
$$y_0 = -\frac{1-e^{-\pi}}{9\pi}$$

- (B) $c_n < 0$ for all $n \ge 1$
- (C) For $n \ge 1$, we have $\frac{|a_n|}{|c_n|} = n^2 + 9$.
- (D) With the same initial conditions, the solution y(x) can be alternatively represented as

$$y(x) = f(x) \star \left(\sum_{n=-\infty}^{\infty} \delta(x - 2n\pi)\right)$$

類組:<u>電機類</u>科目:工程數學 C(3005) ※請在答案卡內作答

共14頁第12頁

where

$$f(x) = \left[\frac{1}{8}\cosh(3x) - \frac{2u(x) - 1}{24}\sinh(3x) - \frac{1}{8}e^{-|x|}\right](u(x + \pi) - u(x - \pi))$$

None of the above is true.



類組: <u>電機類</u> 科目: 工程數學 C(3005)

共14 頁第13 頁

※請在答案卡內作答

十九、 Solve the following boundary value problem for the real-valued bi-variate function f(x,y) for $0 \le x \le 2\pi$ and $y \ge 0$ satisfying

$$\frac{\partial^2 f(x,y)}{\partial x^2} = 4 \frac{\partial^2 f(x,y)}{\partial y^2}$$

$$f(0,y) = f(2\pi,y) = f(x,0) = 0, \left. \frac{\partial f(x,y)}{\partial y} \right|_{y=0} = [\sin(x)]^3.$$

Which of the following statements are true?

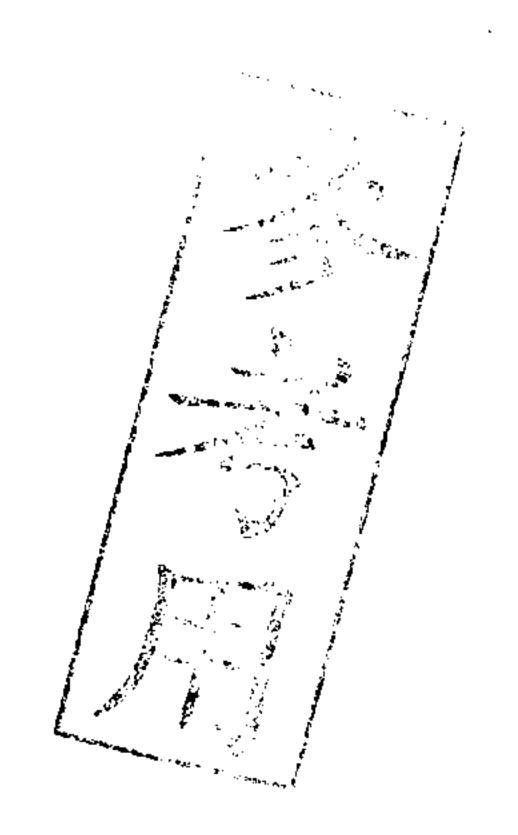
(A)
$$f\left(\frac{\pi}{2},\pi\right)=\frac{4}{3}$$
.

(B)
$$f(\frac{\pi}{2}, \frac{\pi}{2}) = -\frac{5\sqrt{2}}{6}$$
.

(C)
$$\frac{\partial f(x,y)}{\partial y}\Big|_{x=y=\frac{\pi}{2}} = \frac{\sqrt{2}}{4}$$

(D)
$$\frac{\partial^2 f(x,y)}{\partial y^2}\Big|_{x=y=\frac{\pi}{2}} = \frac{3\sqrt{2}}{8}$$

(E) None of the above is true.



共14頁第14頁

※請在答案卡內作答

二十、 Continued from Question 十九, if we change the boundary value problem to

$$\frac{\partial^2 f(x,y)}{\partial x^2} + 4 \frac{\partial^2 f(x,y)}{\partial y^2} = 0$$

while keeping all the boundary- and initial conditions the same for f(x,y), which of the following statements are true?

(A)
$$\frac{\partial f(x,y)}{\partial y}\Big|_{x=\frac{\pi}{4},y=0} = \frac{\sqrt{2}}{2}$$

(B)
$$\frac{\partial f(x,y)}{\partial y}\Big|_{x=\frac{\pi}{2},y=0}=1$$

(C)
$$\frac{\partial^2 f(x,y)}{\partial x^2}\Big|_{y=0} = 1$$

(D)
$$\lim_{y\to\infty} \frac{\ln\left(\left|f\left(\frac{3\pi}{4},y\right)\right|\right)}{y} = \frac{3}{2}$$
.

(E) None of the above is true.

