台灣聯合大學系統104學年度碩士班招生考試試題 共2 頁第 1 頁

類組:電機類 科目:工程數學 A(3003)

※請在答案卷內作答

Note: Detailed derivations are required to obtain a full score for each problem.

1. (15 pts) Let
$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 3 & 5 & a \end{pmatrix}$$
, $x \in M_{3\times 1}(\mathbb{R})$, and $b = \begin{pmatrix} 100 \\ 200 \\ c \end{pmatrix}$.

- (a) (5%) Find the conditions for a such that the system of equations Ax = b has a unique solution.
- (b) (5%) Find the conditions for a and c such that Ax = b has infinitely many solutions.
- (c) (5%) If Ax = b has infinitely many solutions, is it possible to find a positive integer n such that $A^nx = b$ has a unique solution? Why or why not?
- 2. (10 pts) Let V be the vector space spanned by the set of functions $\{1, \cos \omega t, \sin \omega t\}$, defined on the time domain $t \in \mathbb{R}$. Assume that the angular frequency $\omega \geq 0$, and let $\beta = \{1, \cos \omega t, \sin \omega t\}$ be regarded as an ordered basis for V. Define a linear transformation $T: V \to V$ as follows,

$$\mathsf{T}\big(x(t)\big) = m\frac{d^2x(t)}{dt^2} + r\frac{dx(t)}{dt} + kx(t),$$

where parameters m, r, k are non-negative.

- (a) (5%) Find the matrix representation $A = [T]_{\beta}$.
- (b) (5%) If $r = 0, m \neq 0$, find the condition for ω such that dim (N(T)) > 0.
- 3. (9 pts) Let V be an inner product space and let T be a linear operator on V. Prove or disprove the following statement.

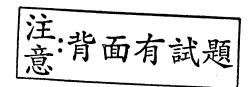
$$R(T^*)^{\perp} = N(T).$$

4. (16 pts) Given the Schur decomposition theorem as follows.

Theorem 1 Let T be a linear operator on a finite-dimensional inner product space V. If $det(T-tI_V)$ splits, then there exists an orthonormal basis β for V such that $[T]_{\beta}$ is upper triangular.

Use this theorem to prove or disprove the following statements.

(a) (8%) Let T be a normal operator on a finite-dimensional inner product space V over \mathbb{C} . Then there exists an orthonormal basis β for V such that $[T]_{\beta}$ is diagonal.



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- (b) (8%) Let T be a self-adjoint operator on a finite-dimensional inner product space V over \mathbb{R} . Then there exists an orthonormal basis β for V such that $[T]_{\beta}$ is diagonal.
- 5. (20 pts) Based on the descriptions below:
 - 1. Two water tanks, denoted as T_1 and T_2 , are mutually connected through two pipelines.
 - 2. Initially tanks T_1 and T_2 contain 100 liters of water each.
 - 3. In tank T_1 water is pure; while 150 grams of salt are dissolved in tank T_2 .
 - 4. By stirring to keep the mixture uniform and circulating liquid through two pipelines at a rate of 2 liters per minute, the amounts of salt $y_1(t)$ in T_1 and $y_2(t)$ in T_2 change with time t.

Write down the differential equations for these two mixing tanks, and How long the tank T_1 will contain at least half as much salt as there will be left in tank T_2 ?

6. (10 pts) Construct a Fourier series over the interval of $(-\pi, \pi)$ and use Parseval identity to calculate the summation of an infinite series:

$$\sum_{n=0}^{\infty} \frac{1}{(2n+1)^4}.$$

- 7. (20 pts) Complex Variables:
 - (a) (5%) Prove or disapprove that e^{iz} is an entire function.
 - **(b)** (5%) Is $Re[\oint f(z) dz] = \oint Re[f(z)] dz$? Explain.
 - (c) (5%) Given that $Ln(1+z) = z \frac{z^2}{2} + \frac{z^3}{3} + \cdots$ for |z| < 1, find the **first three** terms of the Taylor series of Ln(2z) with the center at -2i. Also find its radius of convergence.
 - (d) (5%) Evaluate

$$\oint_C \frac{\exp(-z)}{\cos(z)} \, dz,$$

C: |z| = 2, counter clockwise.

注:背面有試題