1. A time harmonic current source, $\mathbf{J}(\mathbf{R}', t)$, of angular frequency ω and located near the origin, O, is radiating electromagnetic wave away from the source (into free space), as shown in Fig. 1. Eric, an engineer, is asked to find the spatial temporal dependence of the electromagnetic field at the position $\mathbf{R} = (R, \theta, \phi) = (R, \pi/4, \pi/4)$, which is in the *far field zone* of the source.

After analyzing the problem, he shows to you, his supervisor, that the spatial temporal dependence of the electromagnetic field at $\mathbf{R}$ is given by

$$\begin{cases} \mathbf{E}(\mathbf{R},t) = \mathbf{a}_R E_0 R \cos(\beta/R + \omega t), \\ \mathbf{H}(\mathbf{R},t) = \mathbf{a}_\theta \eta_0 E_0 R \sin(\beta/R + \omega t) \end{cases}$$

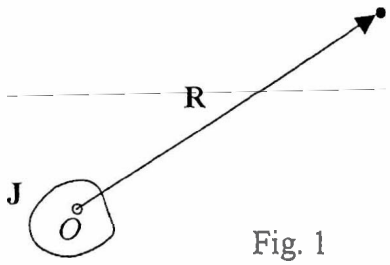


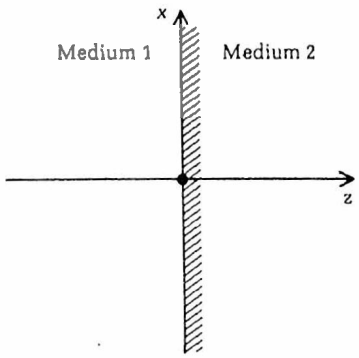
Fig. 1

where $\beta = \omega/c$ is the free space propagation constant and $\eta_0 = \sqrt{\mu_0/\varepsilon_0} \cong 120\,\Omega$. As the supervisor, you are responsible to check the results he obtained. How do you verify his results based on the basic physics laws and properties of the time harmonic electromagnetic waves? Is Eric correct or wrong? Where and **why**, if wrong? Can you write down the $\mathbf{E}(\mathbf{R}, t)$ and $\mathbf{H}(\mathbf{R}, t)$ in a form which is consistent with the basic physics laws and properties of the time harmonic electromagnetic waves? (15%)

2. (a) Consider, for example, a pair of closely spaced parallel conductors such as those electric power cords of small household electric appliances. Would you use the transmission line model, i.e., distributed elements model, to analyze the 60 Hz voltage or current going through the line? What would be the criteria that one needs to use the transmission line model instead of the basic electric circuit theory which is based on the lumped elements model? **Why? (5%)**

(b) For a lossless transmission line terminated with an *open* load, qualitatively draw the voltage and current standing wave pattern along the line. **Explain your results.** Note that you need to label the distance (or spacing, in unit of wavelength) between the maximums and minimums of the standing wave patterns. (5%)

3. A uniform plan wave is incident from medium 1 to medium 2 as shown. The incident wave in medium 1 has amplitude equal to 1 V/m.



Match the following descriptions to the standing wave patterns shown in the figure below and explain briefly the reasoning of your choice. Note: No credit will be given without correct explanation and there are 3 patterns that do not fit any of the following description.

(i) Plot of $|E_{y,total}|$. Medium 1 is air and medium 2 is dielectric characterized by $\varepsilon_2 = 4\varepsilon_0$ and $\mu_2 = \mu_0$. Normal incident. (4%)

(ii) Plot of $|E_{y,total}|$. Medium 1 is dielectric characterized by $\varepsilon_1 = 4\varepsilon_0$ and $\mu_1 = \mu_0$ and medium 2 is air. Normal incident. (4%)

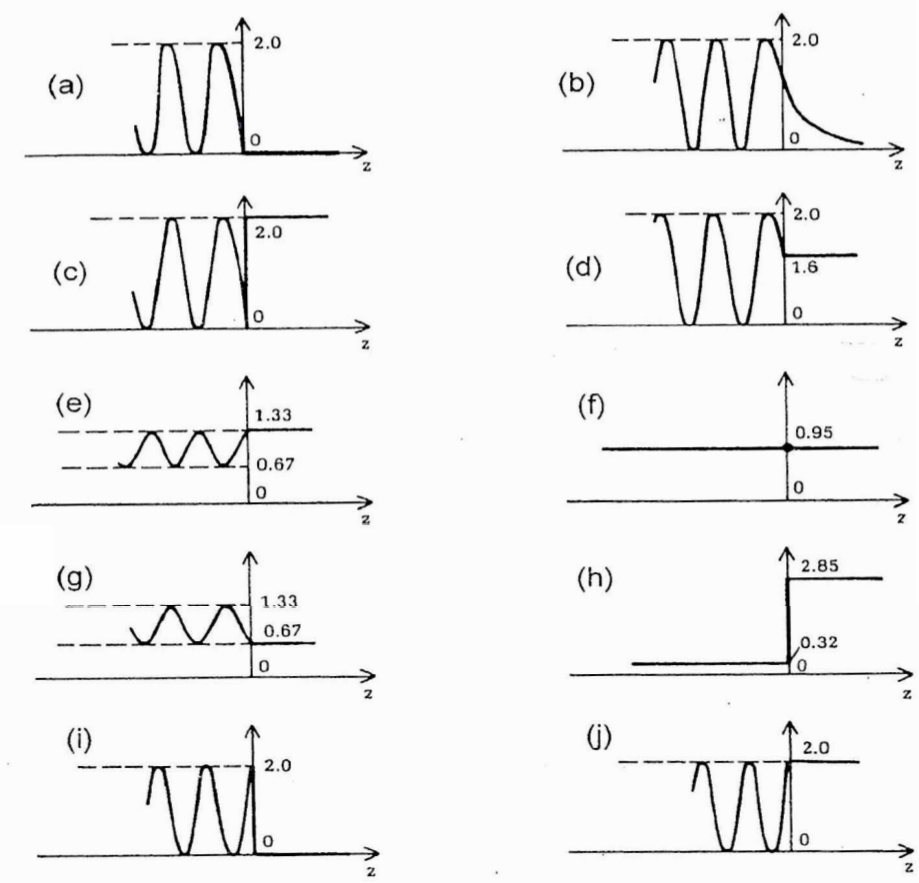
(iii) Plot of $|E_{y,total}|$. Medium 1 is dielectric characterized by $\varepsilon_1 = 4\varepsilon_0$ and $\mu_1 = \mu_0$ and medium 2 is air. The incident angle is greater than the critical angle. (4%)

(iv) Plot of $|E_{x,total}|$. The incident angle is equal to the Brewster angle. (4%)

(v) Plot of $|E_{z,total}|$. The incident angle is equal to the Brewster angle. $\varepsilon_1 > \varepsilon_2$. (3%)

(vi) Plot of $|E_{y,total}|$. Medium 1 is air and medium 2 is perfect conductor. (3%)

(vii) Plot of $|H_{y,total}| \cdot \eta_1$, where η_1 is the intrinsic impedance of medium 1. Medium 1 is air and medium 2 is perfect conductor. (3%)



4. Consider a coaxial cable. Please draw the schematic diagram of vector potential along the signal guided direction. To get credits, you are also asked to explain the physics of your result. (5%)

5. Quarter-wave linear antenna is simple and popular in our broadcasting business. Please schematically draw such an antenna and its far-zone radiation pattern. (5%)

6. Please derive the characteristic impedance of a hollow coaxial cable with the inner and outer diameters a and b, respectively. (15%)

7. The continuity equation relates the current density $\mathbf{J}(\mathbf{r},t)$ with the charge density $\rho(\mathbf{r},t)$. This relation can be given in terms of either the divergence or the flux of the current density. Write down both of these formulas for the continuity equation and explain its physical meaning. (10%)

8. The electric field $\mathbf{E}(\mathbf{r},t)$ can be expressed explicitly in terms of the electric scalar potential Φ and the magnetic vector potential $\mathbf{A}$. Write down this important expression. Then, based on this expression, derive Faraday's law, which is one of Maxwell's equations and relates the spatial variation of electric field with the temporal variation of magnetic field. (15%)