科目：訊號與系統(500C)　校系所組：清大電機工程學系(乙組、丁組)

一、選擇題 60%

(共有 10 題，每題 6 分，爲多選題，答錯一個選項倒扣 1.5 分，請在答案卡上作答)

1. Consider a discrete-time linear time-invariant system with impulse response $h[n]$. Which of the following statements is (are) true?
   (A) If the system is causal, then it has memory.
   (B) If $h[n]$ is a right-sided sequence, then it is causal.
   (C) A necessary and sufficient condition for the system to be stable is that $h[n]$ be absolutely summable.
   (D) If $h[n]=u[n]$ (the unit-step function), then the system is invertible.
   (6%)

2. Consider a discrete-time signal $x[n]$. Which of the following statements is (are) true?
   (A) If $x[n]$ is non-periodic, then $y[n]=x[2n]$ must be non-periodic.
   (B) If $x[n]$ is periodic, then its even part must be periodic.
   (C) If $x[n]=\cos \alpha n$ with $\alpha \neq 0$, then the signal is periodic when $\alpha$ is a rational number of $2\pi$.
   (D) If $x[n]=\cos(\frac{\pi n}{3})\sin(\frac{\pi n}{5})$, then the signal is non-periodic.
   (6%)

3. Consider the following three continuous-time systems $S_1$, $S_2$, and $S_3$ whose responses to a complex exponential input $e^{j5t}$ are specified as
   $S_1 : e^{j5t} \rightarrow te^{j5t}$　$S_2 : e^{j5t} \rightarrow e^{j5(t-1)}$　$S_3 : e^{j5t} \rightarrow \cos(5t)$
   Which of the following answers is correct?
   (A) $S_1$, $S_2$, and $S_3$ are definitely not LTI.
   (B) $S_1$, and $S_3$ are definitely not LTI.
   (C) $S_2$, and $S_3$ are definitely not LTI.
   (D) $S_1$ is definitely not LTI.
   (6%)

4. Let $x(t)$ be a signal with Nyquist rate $\omega_0$. Which of the following signal has the highest Nyquist rate?
   (A) $x(t)\cos(\omega_0 t)$
   (B) $x(t)+x(t-1)$
   (C) $\frac{dx(t)}{dt}$
   (D) $x^2(t)$
   (6%)

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5. Which of the following statement(s) is(are) correct regarding to a second-order continuous-time system described by the differential equation:

$$\frac{d^2y(t)}{dt^2}+2\zeta w_n\frac{dy(t)}{dt}+w_n^2y(t)=w_n^2x(t)$$

where $x(t)$ denotes the input, $y(t)$ the output, $\zeta$ the damping ratio, and $w_n$ the undamped natural frequency?

(A) According to the value of damping ratio, the impulse response of the second-order system is categorized into three cases: underdamped ($0<\zeta<1$), critically damped ($\zeta=1$), and overdamped ($\zeta>1$).

(B) The step response of the system exhibits both overshoot (i.e., the step response exceeds its final value) and ringing (i.e., oscillatory behavior) in the overdamped case.

(C) The underdamped case gives step response the fastest response (i.e., the shortest rise time) that is possible without overshoot and thus has the shortest settling time in such case.

(D) The magnitude response $|H(jw)|$ of the system has a peak at some frequency near $w_n$ in the underdamped case. The less damping there is in the system, the sharper is the peak in $|H(jw)|$.

(6%)

6. Consider a second-order discrete-time causal LTI system described by the difference equation:

$y[n]-2r\cos\theta y[n-1]+r^2y[n-2]=x[n]$ with $0<r<1$ and $0\leqq\theta\leqq\pi$.

(A) The frequency response for this system is $H(e^{j\Omega})=\dfrac{1}{1-2r\cos\theta e^{-j\Omega}+r^2e^{-j2\Omega}}$, and the corresponding impulse response is $h[n]=r^n\dfrac{\sin[(n+1)\theta]}{\sin\theta}u[n]$ for $\theta\neq 0$ or $\pi$.

(B) The impulse response has a damped oscillatory behavior and the step response exhibits ringing and overshoot for any value of $\theta$ other than zero or $\pi$.

(C) The magnitude response $|H(e^{j\Omega})|$ of the system has a peak at some frequency $\Omega$ near $\theta$. The closer to 0 the value of $r$ is in the system, the sharper is the peak in $|H(e^{j\Omega})|$.

(D) If one wishes to use this system to boost the frequency component of an input signal $x[n]$ near $0.25\pi$, the $(0.9, 0.25\pi)$ is a better choice than $(0.1, 0.25\pi)$ for parameters $(r, \theta)$ in this case.

(6%)

7. Determine which of the following statements is(are) true?

(A) The continuous-time signal $x(t) = u(t + T_0) - u(t - T_0)$ can undergo impulse-train sampling without aliasing, provided that the sampling period $T < 2T_0$.

(B) The continuous-time signal x(t) with Fourier transform $X(jw) = u(w + w_0) - u(w - w_0)$ can undergo impulse-train sampling without aliasing, provided that the sampling period $T < \pi/w_0$.

(C) The discrete-time signal $x[n] = u[n + N_0] - u[n - N_0]$ can undergo sampling sequence $p[n]=\sum_{k=-\infty}^{\infty}\delta[n-kN]$ sampling without aliasing, provided that the sampling period $N < 2N_0$.

(D) The discrete-time signal x[n] with Fourier transform $X(e^{j\Omega}) = u(\Omega + \Omega_0) - u(\Omega - \Omega_0)$, $0 < \Omega_0 < \pi$, $|\Omega| < \pi$, can undergo sampling sequence $p[n]=\sum_{k=-\infty}^{\infty}\delta[n-kN]$ sampling without aliasing, provided that the sampling period $N < \pi/\Omega_0$.

(6%)

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8. For the following system,

$$h[n] = 10(-1/2)^n u[n] - 9(-1/4)^n u[n]$$

which of following statements is (are) correct?

(A) This system is a causal and stable system

(B) This system can not be a causal and unstable system

(C) Its inverse system is a causal and stable system

(D) Its inverse system can not be a causal and stable system

(6%)

9. The z-transform of a discrete-time signal $x[n]$ is given by

$$X(z) = \frac{30z^3 - 16z^2 + 2z + 3}{6z^3 - z^2 - z}, \quad ROC: \frac{1}{2} < |z|$$

Which of following answers is (are) correct?

(A) $x[n] = -3\delta[n-1] + \delta[n] - 3\delta[n+1] - 21\delta[n+2] + 3\delta[n+3] + \cdots$

(B) $x[n] = 5\delta[n] - \frac{11}{6}\delta[n-1] + \frac{31}{36}\delta[n-2] + \frac{73}{216}\delta[n-3] + \cdots$

(C) $x[n] = \delta[n] - 3\delta[n-1] + (\frac{-1}{3})^n u[n] + 3(\frac{1}{2})^n u[n]$

(D) $x[n] = 5\delta[n] - 3\delta[n-1] + \frac{3}{2}(\frac{1}{2})^{n-1} u[n-1] - \frac{1}{3}(\frac{-1}{3})^{n-1} u[n-1]$

(6%)

10. The bilateral Laplace transform of a continuous-time signal $x(t)$ is specified by,

$$X(s) = \frac{s+4}{(s+2)(s^2+6s+13)} \quad \text{with ROC}: -3 < \text{Re}(s) < -2$$

Which of following answers is (are) correct?

(A) $x(t) = \frac{2}{5}e^{-2t}u(t) - \frac{2}{5}e^{-3t}\cos(2t)u(t) + \frac{3}{10}e^{-3t}\sin(2t)u(t)$

(B) $x(t) = -\frac{2}{5}e^{-2t}u(-t) - \frac{2}{5}e^{-3t}\cos(2t)u(t) + \frac{3}{10}e^{-3t}\sin(2t)u(t)$

(C) $x(t) = -\frac{2}{5}e^{-2t}u(-t) + \frac{2}{5}e^{-3t}\cos(2t)u(t) - \frac{3}{10}e^{-3t}\sin(2t)u(t)$

(D) $x(t) = \frac{2}{5}e^{-2t}u(t) + \frac{2}{5}e^{-3t}\cos(2t)u(-t) - \frac{3}{10}e^{-3t}\sin(2t)u(-t)$

(6%)

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二、演算題 40%

(共有4題，每題10分，請在答案卷上作答，記得註明作答題號)

11. Consider a linear time-invariant system with impulse response $h(t) = e^{-t}u(t+1)$. Determine the output $y(t)$ of the system when the input is $x(t) = \sin^2 t$.
(10%)

12. The Parseval's relation states that the energy of a discrete-time signal can be determined by integrating the energy per unit frequency over a full $2\pi$ interval of distinct discrete-time frequencies. . Using Parseval's relation, evaluate the following integral:

$$\int_0^{\pi} \frac{4}{5+4\cos\omega} d\omega$$

(10%)

13. Suppose that $x[n]$ has a Fourier transform $X(e^{j\Omega})$ that is zero for $\pi/3 \leqq |\Omega| \leqq \pi$. If $x[n]$ is first sampled by a sampling sequence $p[n]$ ( $p[n] = \sum_{k=-\infty}^{\infty} \delta[n-kN]$ ) with sampling period $N$ (sampling frequency $\Omega_s = 2\pi/N$) to obtained a sampled sequence $x_p[n] = x[n]\,p[n]$, and then filtered by a discrete-time ideal lowpass filter $H(e^{j\Omega})$ with cutoff frequency $\Omega_s/2$ and gain $N$ to obtain a output sequence $x_r[n]$,
(a) give the maximum value of $N$ such that $X_r(e^{j\Omega}) = X(e^{j\Omega})$, and under such $N$ (2%)
(b) derive a formula for $X_r(e^{j\Omega})$ in terms of $X(e^{j\Omega})$, (4%)
(c) derive a formula which expresses $x_r[n]$ in terms of samples of $x[n]$ in this case. (4%)

14. Determine the continuous-time signal corresponding to the following unilateral Laplace transform,

$$X(s) = s\frac{d^2}{ds^2}\left(\frac{1}{s^2+25}\right).$$

(10%)