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96學年度 電機領域聯合招生 系(所) 碩士班入學考試

Note: There are total of 6 problem sets with total of 100 points.

1. (16%) An analog signal $m(t)$ is sampled, quantized, and encoded into a binary PCM signal. The PCM system has 256 representation levels for each sample. The PCM wave is transmitted over a band-limited 64 kHz baseband channel using quaternary PAM system with raised-cosine spectrum. The roll-off factor α of the raised-cosine spectrum is 1 to avoid inter-symbol interference at the expense of excess channel bandwidth over the ideal solution.
   (a) (4%) Suppose you are asked to design this baseband transmission system, could you draw the block diagram the system with Quantizer, Raised-Cosine Channel, PAM, Line Encoder, and Sampler?
   (b) (4%) Find the bit rate (bits/sec) at which the information is transmitted through the channel.
   (c) (4%) Find the rate at which the analog signal $m(t)$ is sampled.
   (d) (4%) What is the max possible value for the highest frequency component of the analog signal such that aliasing effect would not occur during the sampling process?

2. (18%) Suppose $x[n]$'s are discrete time samples from the sample function of $X(t)$ which is a stationary zero-mean random process. The correlation of $x[n]$ is $\vec{r}_x = [R_x[1], R_x[2], \ldots, R_x[p]]^T$, where $R_x[k] = E\{x[n]x[n-k]\}$. We want to predict $x[n]$ using the its previous values, $x[n-1], x[n-2], \ldots, x[n-p]$ by

$$\hat{x}[n] = \sum_{k=1}^{p} w_k x[n-k]$$

   We want to find the coefficient $w_1, w_2, \ldots, w_p$ such that the error $e[n] = x[n] - \hat{x}[n]$ has minimum power, i.e. $J = E\{e^2[n]\}$ is minimized. This is called the minimum mean square error (MMSE) estimation.
   (a) (5%) Determine the optimum coefficient $\vec{w}_{opt} = [w_1, w_p, \ldots, w_p]^T$ such that J is minimized.
   (b) (4%) Find the minimum J, i.e. J_{min}.
   (c) (5%) Let the sinusoidal signal with random phase $X(t) = A\cos(2\pi f_0 t + \theta)$, where A and f_0 are constant and θ is a random variable that uniformly distributed over the interval $[-\pi, \pi]$. The sampling rate is $8f_0$, and the predictor has a single coefficient w_1. Please determine the optimum value of w_1 to minimize J.
   (d) (4%) Please determine the J_{min}.

3. (20%) Let X be a continuous random variable uniformly distributed in the interval $[-2, 2]$, i.e. the probability density function (pdf) of X is

$$f_X(x) = \frac{1}{4} \quad \text{for } x \in [-2, 2]$$

and $f_X(x) = 0$, otherwise. Suppose that the random variable X is encoded into the symbol U and the signal received at the destination is given by

$$Y = U + N$$

where N is Gaussian with mean 0 and variance σ^2.

(a) (6%) Suppose that $U = \alpha X$ and the estimate at the receiver is $\hat{X} = Y / \alpha$. Compute the value of α such that $E[U^2] = 1$ and calculate the mean square error of the estimate, i.e. $E[(\hat{X} - X)^2]$.

(b) (10%) Suppose a binary quantization is performed at the transmitter such that

$$U = \begin{cases} 1 & X \in [0, 2] \\ -1 & X \in [-2, 0). \end{cases}$$

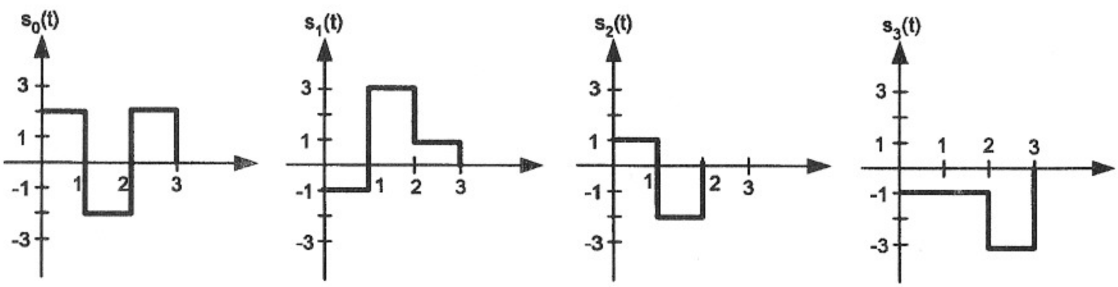
Based on the received signal Y, the estimate at the receiver is made such that

$$\hat{X} = \begin{cases} 1 & Y \geq 0 \\ -1 & Y < 0. \end{cases}$$

Compute the mean square error of the estimate.

(c) (4%)When the noise variance σ^2 is large, would you choose the scheme in (a) or the scheme in (b)? Why?

4. (14%) Consider the 4 signal waveforms $s_0(t)$, $s_1(t)$, $s_2(t)$ and $s_3(t)$ as shown in the following



(a) (10%) Use the Gram-Schmidt procedure (starting from $s_0(t)$ and followed in order by $s_1(t)$, $s_2(t)$ and $s_3(t)$) to find the orthonormal basis functions for the 4 signal waveforms shown in the figure. Plot each of the orthonormal basis functions that you obtained.

(b) (4%) Give the vector representation of the 4 signal waveforms corresponding to the orthonormal basis functions obtained in (a).

5. (24%) In this problem, we consider a digital pass-band transmission system using an optimum design of the receiver in the sense that it will make fewer errors in the long run than any other receiver in an additive white noise channel with double-sided power spectral density $N_0/2$. No *a priori* information is available in this transmission system. Let E_b denote the energy per message bit. First, consider the error performance for the following signaling (modulation) schemes

(i) (2%) Non-coherent binary FSK (frequency-shift keying)
(ii) (2%) Coherent binary PSK (phase-shift keying)
(iii) (2%) Coherent MSK (minimum shift keying)
(iv) (2%) Conventional coherent binary FSK with one-bit decoding
(v) (2%) Coherent QPSK (quadriphase-shift keying)
(vi) (2%) Differentially coherent detection of binary DPSK (differential phase-shift keying)

For each signaling (modulation) scheme, please choose the correct expression of bit error rate (BER) from the candidates given as follows. (The complementary error function $erfc(u)$ is defined by $erfc(u) = 1 - \frac{2}{\sqrt{\pi}} \int_0^u - z^2 dz$.)

(a) $\frac{1}{2} erfc\left(\sqrt{E_b/N_0}\right)$ (b) $erfc\left(\sqrt{E_b/N_0}\right)$ (c) $\frac{1}{2} erfc\left(\sqrt{E_b/2N_0}\right)$ (d) $erfc\left(\sqrt{E_b/2N_0}\right)$

(e) $\frac{1}{2} erfc\left(\sqrt{2E_b/N_0}\right)$ (f) $erfc\left(\sqrt{2E_b/N_0}\right)$ (g) $\frac{1}{2}\exp\left(-E_b/N_0\right)$ (h) $\exp\left(-E_b/N_0\right)$

(i) $\frac{1}{2}\exp\left(-E_b/2N_0\right)$ (j) $\exp\left(-E_b/2N_0\right)$ (k) $\frac{1}{2}\exp\left(-2E_b/N_0\right)$ (l) $\exp\left(-2E_b/N_0\right)$.

(3%) Please plot BER as a function of the signal energy per-bit to noise spectral density ratio, E_b/N_0, for the signaling (modulation) schemes described in (i) to (vi).

Now consider the power spectra for the following signaling (modulation) schemes.
(vii) (2%) Binary FSK (frequency-shift keying)
(viii) (2%) Binary PSK (phase-shift keying)
(ix) (2%) QPSK (quadriphase-shift keying)

Let $S_B(f)$ denote the power spectral density of the complex envelope of the pass-band signal and T_b be the bit duration. For each signaling (modulation) scheme (vii-ix), please choose the correct expression of $S_B(f)$ from the candidates given as follows.

(a) $\frac{2E_b \sin^2(2\pi T_b f)}{(2\pi T_b f)^2}$ (b) $\frac{4E_b \sin^2(2\pi T_b f)}{(2\pi T_b f)^2}$ (c) $\frac{E_b}{4T_b}\left[\delta\left(f - \frac{1}{T_b}\right) + \delta\left(f + \frac{1}{T_b}\right)\right] + \left|\frac{8E_b \cos(2\pi T_b f)}{\pi^2(16T_b^2 f^2 - 1)}\right|$

(d) $\left|\dfrac{2E_b\sin(\pi T_b f)}{(\pi T_b f)}\right|$ (e) $\left|\dfrac{4E_b\sin(2\pi T_b f)}{(2\pi T_b f)}\right|$ (f) $\dfrac{E_b}{2T_b}\left[\delta\left(f-\dfrac{1}{2T_b}\right)+\delta\left(f+\dfrac{1}{2T_b}\right)\right]+\dfrac{8E_b\cos^2(\pi T_b f)}{\pi^2(4T_b^2f^2-1)^2}$

(g) $\dfrac{2E_b\sin^2(\pi T_b f)}{(\pi T_b f)^2}$ (h) $\left|\dfrac{2E_b\sin(2\pi T_b f)}{(2\pi T_b f)}\right|$ (i) $\dfrac{E_b}{4T_b}\left[\delta\left(f-\dfrac{1}{T_b}\right)+\delta\left(f+\dfrac{1}{T_b}\right)\right]+\dfrac{8E_b\cos^2(2\pi T_b f)}{\pi^2(16T_b^2f^2-1)^2}$

(j) $\dfrac{E_b}{2T_b}\left[\delta\left(f-\dfrac{1}{2T_b}\right)+\delta\left(f+\dfrac{1}{2T_b}\right)\right]+\left|\dfrac{8E_b\cos(\pi T_b f)}{\pi^2(4T_b^2f^2-1)}\right|$

(3%) Please plot the normalized power spectral density $S_B(f)/E_b$ versus the normalized frequency fT_b for the signaling (modulation) schemes described in vii-ix.

6. (8 %.) Consider a BPSK system in the additive white noise (AWGN) channel with double-sided power spectral density $N_0/2$ with equally waveforms $s_1(t)=\sqrt{\dfrac{2E_b}{T_b}}\cos(2\pi f_c t)$ and $s_2(t)=-\sqrt{\dfrac{2E_b}{T_b}}\cos(2\pi f_c t)$, where $0\le t\le T_b$ and E_b is the transmitted signal energy per bit. At the matched filter detector, the $s_1(t)$ reference is $\sqrt{\dfrac{2}{T_b}}\cos(2\pi f_c t+\phi)$ $0\le t\le T_b$, where ϕ is the phase error. Determine the effect of the phase error ϕ on the average probability error of the BPSK system.