*** Note: You must give detail derivations, otherwise you get no points.**

1. Consider the quantization process in a PCM (pulse-code modulation) system. The message signal $m(t)$ is sampled and then quantized. The probability density function of the sampled value m is assumed to be $f(m)$.

   (a) If a set of representation levels $\{v_1, v_2, \cdots, v_L\}$ is available, determine the optimal partition cell ϑ_k, $1 < k < L$, based on the mean-square distortion criterion. (5%)

   (b) If a set of partition cells $\{\vartheta_1, \vartheta_2, \cdots, \vartheta_L\}$ exists, determine the optimal representation levels $\{v_1, v_2, \cdots, v_L\}$ based on the mean-square distortion criterion. (5%)

   (c) According to the result obtained in (b), if the probability density function in the partition cell [0, 1) is $f(m) = 0.12m^2 + 0.06m + 0.1$, find the optimal representation level in this cell. (5%)

2. Consider a linear two-port device whose input resistance is matched to the internal resistance, denotes as R_s, of the source. The available power gain of the two-port device is assumed to be G. The mean-square value of this noise voltage is $4kTR_s\Delta f$, where k is Boltzmann's constant, T is the room temperature and Δf is the signal bandwidth.

   (a) Find the input noise power P_i. (2%)

   (b) Assume that the noise figure of this device is F and the noise power contributed by this device is $P_d = GkT_e\Delta f$, where T_e is the equivalent noise temperature. Determine the relation between T_e and F. (3%)

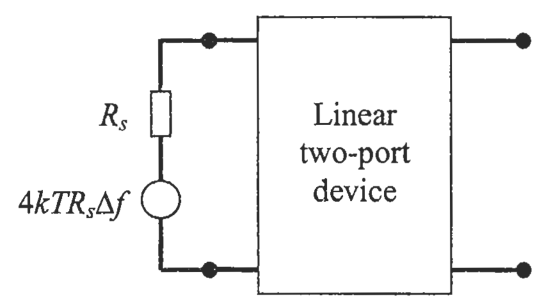


Fig. P2-1

   (c) Assuming that the room temperature is T=290° K, find the equivalent noise temperature for each of the three components shown in Fig. P2-2. The antenna equivalent noise temperature is 50° K. (5%)

   (d) Find the effective noise temperature of the whole receiver. (5%) [Hint: the noise figure of a cascade connection of any number of noisy two-port networks is $F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1G_2} + \frac{F_4 - 1}{G_1G_2G_3} + \cdots$]

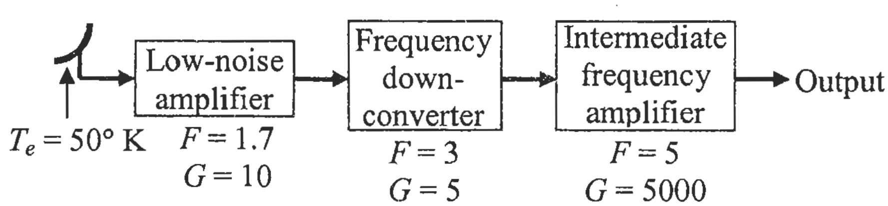


Fig. P2-2

3. The message $m(t)$, whose spectrum is shown in Fig. P3, is passed through the system shown in the same figure. The bandpass filter has a bandwidth of $2W$ centered at f_0, where $f_0 >> W$, and the lowpass filter has a bandwidth of W.

   (a) Plot the spectra of the signals $x(t)$, $y_1(t)$, $y_2(t)$, $y_3(t)$ and $y_4(t)$. (10%)

   (b) What are the bandwidths of these signals? (10%)

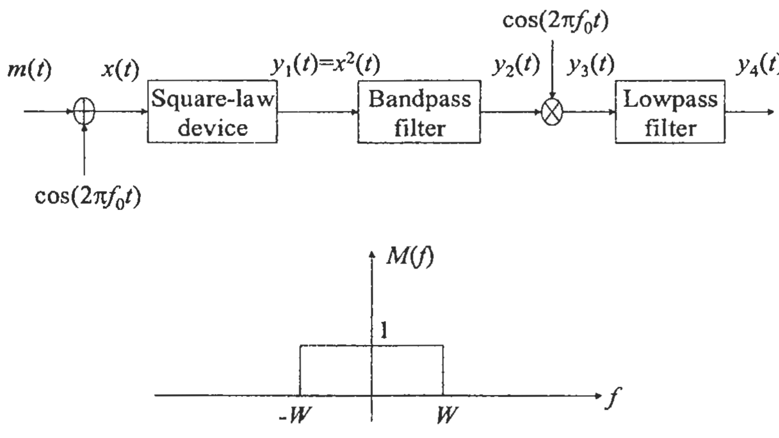


Fig. P3

4. In a binary antipodal signaling scheme, the signals are given by

$$s_1(t) = -s_2(t) = \begin{cases} A, & 0 < t < T_b \\ 0, & \text{otherwise} \end{cases}$$

   The channel is an additive white Gaussian noise (AWGN) channel with the noise power spectral density $S(f) = N_0/2$. The two signals $s_1(t)$ and $s_2(t)$ have prior probabilities p_1 and $p_2 = 1 - p_1$, respectively.

   (a) Determine the structure of the optimal receiver, including the threshold expressed as a function of A, T_b, N_0 and p_1. (7%)

   (b) Determine an expression for the average error probability. (8%)

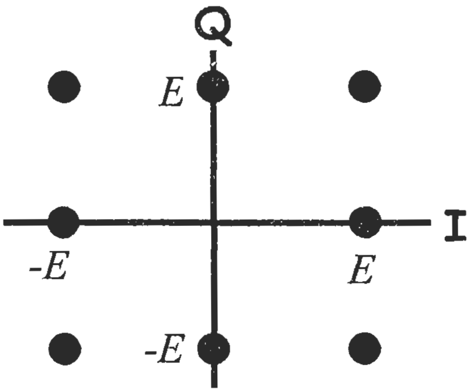
5. Consider the two 8-point QAM constellations A and B and the distance between adjacent points as shown in Fig. P5.

   (a) Please determine the symbol rate if the desired bit rate is 54Mbps. (5%)

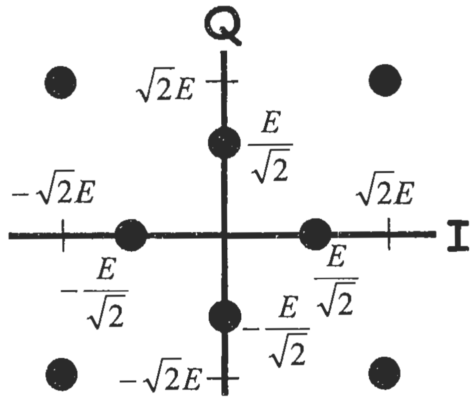
   (b) Please determine the average transmitted power for each constellation assuming that all signal points are equally probable. (3%)

(c) Which constellation is more power efficient? (2%)

(d) Compare the SNR (signal-to-noise ratio) required for the two constellations having the same error probability. (5%)



(a) Constellation A



(b) Constellation B

Fig. P5. 8-point QAM constellations

6. Consider a linear channel being divided into 3 subchannels with frequency bands (0, W_1), (W_1, W_2), and (W_2, W). Each channel has squared magnitude response $|H(f)|^2$ in the piecewise-linear form with $|H(f_n)|^2=$ 1, 1/3 and 1/9 for subchannels n = 1, 2, and 3, respectively. Assume the system transmits data at the rate equal to the Shannon's channel capacity and the noise variance σ_n^2 = 1, 1/2 and 1/4 for subchannels n = 1, 2, and 3, respectively. [Hint: According to Shannon's information capacity theorem, the capacity of an AWGN channel is defined by $C = B\log_2(1+SNR)$ bps, where B is the channel bandwidth and SNR denotes the signal-to-noise ratio at the channel output.]

(a) Please derive the formulas for the optimum powers P_1, P_2, and P_3 allocated to these three subchannels such that the overall channel capacity of the entire system can be maximized. (5%)

(b) Given the total transmit power P = 40mW, please calculate the corresponding values of P_1, P_2, and P_3. (5%)

(c) Please use water-filling interpretation sketch (energy vs subchannel frequency) to illustrate the loading problem. (5%)

(d) Suppose this multichannel system is designed to have 64 bits per symbol, and each subchannel is modulated with a QAM modulator. Could you allocate the symbol bits appropriately to each subchannel according to the power allocation you have found in (b) to optimize the channel capacity? (5%)