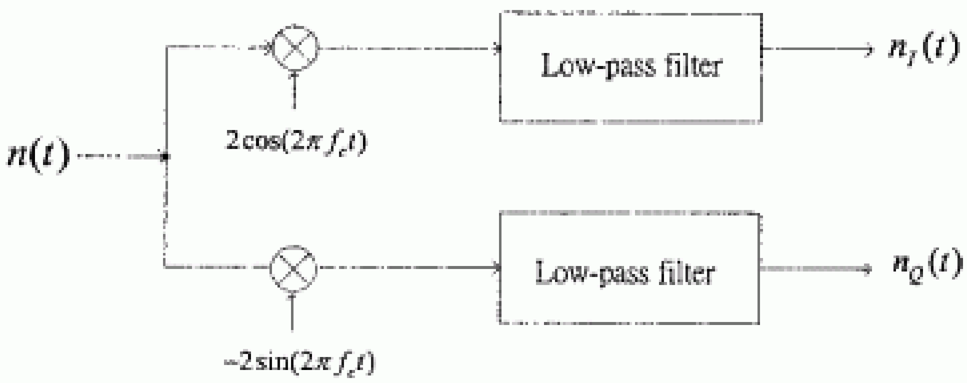
1. A broadband Gaussian signal $s(t)$ of zero mean and power spectral density $S_0/2$ is applied to an ideal low-pass filter of bandwidth B and passband magnitude response of one. Denote the filter output as $v(t)$.
   (a) Find the power spectral density of $v(t)$ and plot it. (4%)
   (b) Find the autocorrelation function of $v(t)$ and plot it. (5%)
   (c) At what sampling rate will the resulting samples of $v(t)$ be statistically independent? (2%)
   What is the mean and the variance of each such sample? (2%)

2. Prove the following statements:
   (a) An autocorrelation function is an even function and has its maximum value at $\tau = 0$. (4%)
   (b) If $X(t)$ contains a sinusoidal component, then the autocorrelation function $R_X(\tau)$ will also contain a sinusoidal component of the same frequency. (4%)
   (c) Both the in-phase component $n_I(t)$ and quadrature component $n_Q(t)$ have the same spectral density, which is related to the power density $S_N(f)$ of the narrowband noise $n(t)$ as

$$S_{N_I}(f) = S_{N_Q}(f) = \begin{cases} S_N(f - f_c) + S_N(f + f_c), & -B \le f \le B \\ 0 & , otherwise \end{cases}$$

   Where it is assumed that $S_N(f)$ occupies the frequency interval $f_c - B \le |f| \le f_c + B$, and $f_c > B$. (8%)

   [Hint: Use the following scheme to extract $n_I(t)$ and $n_Q(t)$:



   where the low-pass filters are ideal with a bandwidth equal to one-half that of the narrowband noise $n(t)$.]

3. A single-sideband amplitude modulated (SSB-AM) signal $s(t)$ is generated by modulating a carrier by the signal $m(t)$. The SSB-AM signal $s(t)$ is given by $s(t) = A_c m(t)\cos(2\pi f_c t) + A_c \hat{m}(t)\sin(2\pi f_c t)$, where $A_c = 10$ is the amplitude of the carrier, $f_c = 500$ kHz is the carrier frequency, and the modulating signal is given by $m(t) = \cos(2000\pi t) + 4\sin(6000\pi t)$.
   (a) Derive the Hilbert transform $\hat{m}(t)$ of $m(t)$. (5%)
   (b) Is $s(t)$ an upper or lower sideband SSB-AM signal? You must give a proof of your answer. (5%)
   (c) Draw the magnitude spectrum of $s(t)$. (5%)

4. Consider the problem of binary signal transmission over an additive white Gaussian noise (AWGN) channel specified by $r = s + n$, where r is the received signal, $s \in \{s_0, s_1\}$ $(s_0 < s_1)$ is the transmitted signal, and $n \sim N(0, \sigma^2)$ is the additive Gaussian noise. Assume that $\Pr\{s = s_0\} = p_0$ and $\Pr\{s = s_1\} = p_1$.
   (a) Derive the optimal decision rule that minimizes the probability of error. (5%)
   (b) In fact, the optimal decision rule in (a) compares the received signal r with a threshold τ. What is τ when $p_0 = p_1$? How does τ change as the prior probability p_0 increases from 0 to 1 when the noise variance σ^2 is finite? How does τ change as the noise variance σ^2 increases from 0 to ∞ when $0 < p_1 < p_0$ and when $0 < p_0 < p_1$? (5%)
   (c) Derive the minimum probability of error P_e. (5%)
   (d) What is P_e when $p_0 = p_1$? What is P_e when $\sigma^2 = 0$? What is P_e when $\sigma^2 = \infty$ and $0 < p_1 < p_0$? What is P_e when $\sigma^2 = \infty$ and $0 < p_0 < p_1$? (5%)

5. (a) Briefly explain the meanings and importance of carrier recovery and symbol timing recovery. (6%)
   (b) Take a coherent QPSK system as an example. Derive the functional blocks for carrier recovery and symbol timing recovery, respectively. Also discuss the effects of the derived blocks under different E_b/N_0. (10%)

6. Consider a direct sequence spread spectrum system.

(a) Let the message be $b(t)$, spreading sequence be $c(t)$, and carrier frequency be $\frac{W_c}{2\pi}$. Draw a block diagram of the system and explain how the system has anti-jamming capability. (10%)

(b) Let the PN code rate be 192×10^6 chips per second and a binary message bit rate at 7500 bps. If QPSK instead of BPSK, what is the processing gain? (5%)

(c) Assuming the received signal power is 8×10^{-14} watts and the one-side noise spectral density level, N_0, is $1.6\times10^{-20}\,W/_{HZ}$, find the signal-to-noise power ratio in the input of the receiver. (Use the information from (b)). (5%)