國 立 清 華 大 學 命 題 紙

九十二學年度 通訊工程／電機工程研究所 甲／乙 組碩士班研究生招生考試

科目 通訊系統 科號 3002／2404 共 3 頁第 1 頁 *請在試卷【答案卷】內作答

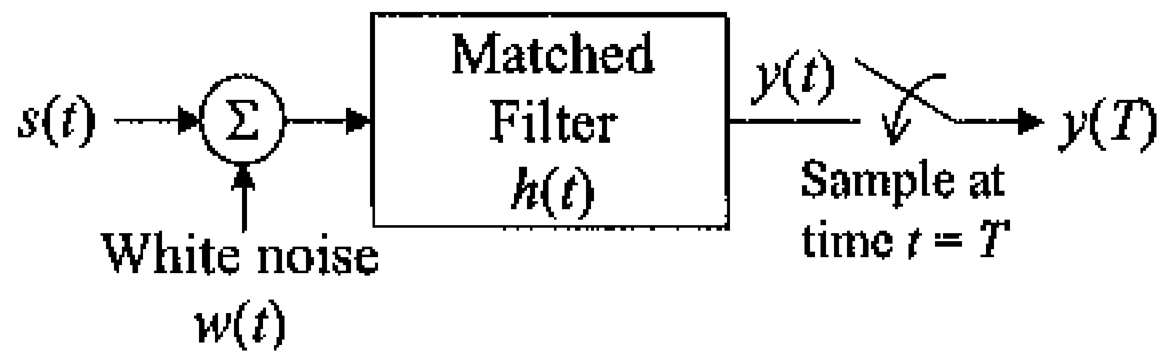
1. A message signal with bandwidth equal to 4.7 kHz and amplitude in the range of (-10, 10) volts is transmitted via a PCM system. The maximum acceptable quantization error is less than 0.5 volt and the sampling rate is $R_s \times 10^3$ samples/sec, where R_s is an integer.

   (a) What is the minimum required bandwidth (in bps) for the transmission? (5%)

   (b) If the quantization error is modeled as a thermal noise, and the transmission bandwidth is extended to 80 kbps, what is the gain of output SNR as compared to (a)? (5%)

   [Note: You must give detail derivations, otherwise you get no points.]

2. A pulse signal $s(t)$, $0 \le t \le T$, with Fourier transform pair $S(f)$ is received by a matched filter $h(t)$, and the output waveform is $y(t)$. The output signal $y(T)$ is obtained by sampling $y(t)$ at $t = T$.



   (a) Determine the frequency response of this matched filter $H(f)$. (5 %)

   (b) Find the output signal $y(T)$. (5 %)

   (c) If the two-sided power spectral density of $w(t)$ is $N_0/2$, find the output SNR. (5 %)

   [Note: Express the answers in terms of $S(f)$. You must give detail derivations, otherwise you get no points.]

3. A white Gaussian noise $w(t)$ with zero mean and two-sided power spectral density $N_0/2$ is filtered by an ideal band-pass filter with passband magnitude response equal to one, midband frequency f_c, and bandwidth $2B$. The filtered noise is denoted as $n(t)$.

   (a) According to the power spectral density of $n(t)$, find the autocorrelation function of $n(t)$. (5 %)

   (b) The representation of $n(t)$ in terms of its in-phase and quadrature components is $n(t) = n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$. According to the result of (a), find the autocorrelation functions of $n_I(t)$ and $n_Q(t)$. (5 %)

   (c) According to the result of (b), find the power spectral density of $n_I(t)$ and $n_Q(t)$. (5 %)

   [Hint: $\mathrm{sinc}(2Wt) \xleftrightarrow{\text{Fourier Transform}} \frac{1}{2W}\mathrm{rect}\left(\frac{f}{2W}\right)$]

   [Note: You must give detail derivations, otherwise you get no points.]

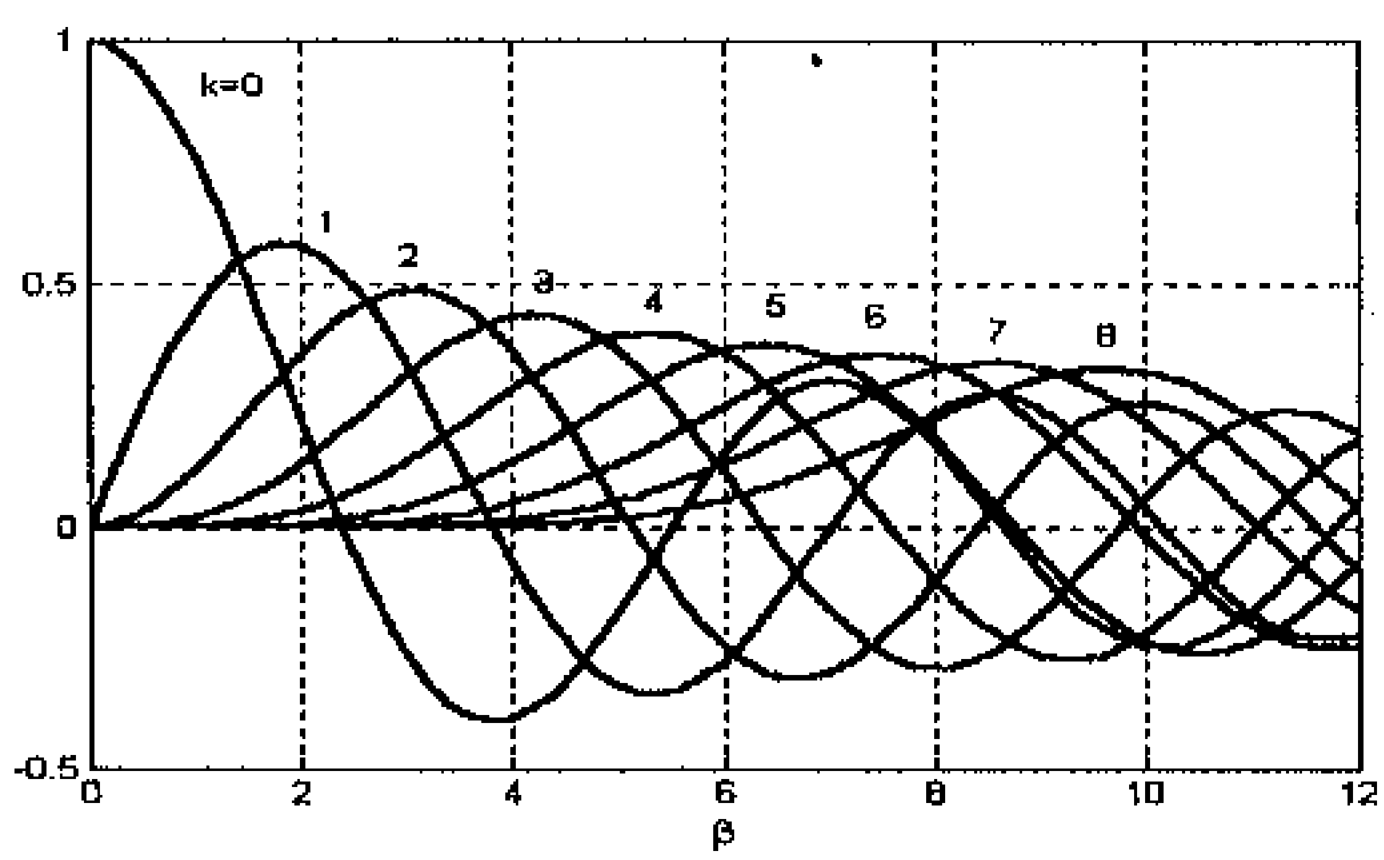
4. A signal's power is defined as $P_s = \lim_{T\to\infty}\left\{\frac{1}{2T}\int_{-T}^{T}|s(t)|^2\,dt\right\}$.

(a) Assume an amplitude modulation signal: $s(t) = A_c(a_0 + m(t))\cdot\cos(2\pi f_c t)$, where $m(t) = A_m\cos(2\pi f_m t)$ is the baseband signal with amplitude A_m and frequency f_m, and A_c and f_c are the carrier's amplitude and frequency, respectively. If no overmodulation for all t, what is the power, P_s, carried by this AM signal? (7 %)

(b) If a carrier with an amplitude, A_c, of 2 Volts is modulated with a sine wave $m(t)$ having a frequency at 10 kHz and a power of 2 Watts, assumed across 1-Ω resistor. The modulated signal is to have a power, P_s, of 10 Watts. What is the power in the carrier component? (3 %)

(c) Same as (a), but the signal is now in frequency modulation format, the signal can be written as $s(t) = A_c\cos(2\pi f_c t + 2\pi\Delta_f\int m(t)dt) = A_c\sum_{-\infty}^{\infty}J_n(\beta)\cos[2\pi(f_c + nf_m)t]$, where Δ_f is the frequency deviation and β is a function of Δ_f. Express β in terms of A_m, f_m, Δ_f and what is the power, P_s, carried by this FM signal? The definition of the first kind Bessel function is $J_n(\beta) = (1/2\pi)\int_{-\pi}^{\pi}\exp[j(\beta\sin(x) - nx)]dx$ (7 %)

(d) Refer to the nth order Bessel function of the first kind plot shown as below, please estimate how much frequency deviation, in terms of f_m, should be if the power of the carrier component is 0? (3 %)



5. (a) If a filter has a unit gain and linear phase function $H(\omega) = \exp(-j\alpha\omega)$, what happens to the input signal? (5 %)

(b) A differentiator has the filter transform function $H(\omega) = j\omega$. If the input is a random process $x(t)$ and output is $y(t)$, what is the relationship between the input and output autocorrelation, $R_x(\tau)$ and $R_y(\tau)$? (5 %)

6. Consider the M-ary Coherent Frequency-Shift Keying (FSK), for which the transmitted signals are defined by

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left[\frac{\pi}{T}(n_c + i)t\right], \quad 0 \le t \le T,$$

where $i = 1, 2, \ldots, M$ and the carrier frequency $f_c = n_c/2T$ for some fixed integer n_c. The transmitted symbols are of equal duration and have equal energy E with equal prior probability $1/M$.

(a) In the presence of additive white Gaussian noise with power spectral density equal to $N_0/2$, design the optimum receiver with minimum probability of symbol error. (8 %)

(b) Find the minimum symbol error rate, P_e for $M = 2$. For $M > 2$, find the union bound for P_e in terms of complementary error function erfc(u) or in terms of Q-function $Q(u)$ where

$$\operatorname{erfc}(u) = \frac{2}{\sqrt{\pi}} \int_u^\infty \exp(-z^2)dz = 2Q(\sqrt{2}u) \text{ and}$$

$$Q(u) = \frac{1}{\sqrt{2\pi}} \int_u^\infty \exp(-z^2/2)dz. \text{ (7 \%)}$$

[Hint: The Gaussian distribution is $f_X(x) = \frac{1}{\sqrt{2\pi}\sigma_X} \exp\left[-\frac{(x-\mu_X)^2}{2\sigma_X^2}\right]$.]

7. Let $s(t)$ denote the signal of coherent binary Phase-Shift Keying (BPSK) signal with energy per bit E_b, bit duration T_b and carrier frequency f_c.

(a) Express $s(t)$ in terms of E_b, T_b and f_c. What is the associated DS/BPSK signal $x(t)$ with the spreading factor N? (7 %)

(b) Design a receiver to detect the data sequence from the received signal $y(t) = x(t) + j(t)$ where $j(t)$ is the interference signal. (8 %)