國 立 清 華 大 學 命 題 紙

九十一學年度 電機 系（所） 丙 組碩士班研究生招生考試

科目 近代物理 科號 2524 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

1. An electron is confined to a one-dimensional box with side L. (a) Calculate the energy of the electron. (b) If the ground state energy of the electron is equal to kT at 300K, what is the value of L?
The value of Boltzmann constant is $k=1.38\times10^{-23}$ J/K. The value of Planck constant is $h= 6.626\times10^{-34}$ J·s. (15%)

2.(a) Find the de Broglie wavelength of a 46-gram golf ball with a velocity of 30 m/s.
(b) Find the de Broglie wavelength of an electron with a velocity of 10^7 m/s.
(c) What does the magnitude of these two wavelengths tell you? (15%)

3. Consider a particle of mass m moving in the following one-dimensional staircase potential

$V(x) = 0$, for $x < -d$,
$= V_0$, for $-d \le x < 0$,
$= \infty$ for $x \ge 0$

where $V_0 > 0$ and $d > 0$.

a) Solve for the wave function of the particle with energy $E > V_0$. Normalization of wave function is not required. (8%)

b) Solve for the wave function of the particle with energy $E < V_0$. Normalization of wave function is not required. (8%)

4. Consider a particle of mass m moving in the following semi-infinite potential barrier

$V(x) = 0$, for $x < 0$
$= V_0$, for $x \ge 0$,

where $V_0 > 0$. Suppose the particle has an incident wave component = exp(ikx), where $k > 0$, and an energy $E = (\hbar k)^2/2m < V_0$.

a) Solve for the wave function inside the barrier. (6%)

b) Calculate the probability current (that is, tunneling current) inside the barrier. (8%)

c) How deep does the particle tunnel into the barrier before its probability decays by a factor e^{-1} relative to the probability at $x = 0$? (5%)

5. The wavefunction of the ground state of a hydrogen atom is $\psi_{100}(r,\theta,\phi)=(\pi)^{-\frac{1}{2}}\left(\frac{1}{a_0}\right)^{\frac{3}{2}} e^{-\frac{r}{a_0}}$

where $a_0 = \frac{\hbar^2}{m_e k e^2}$ is the Bohr radius. (i) Show that the probability of finding the electron anywhere in the spherical shell of radius r and differential thickness dr is $P_{100}(r)dr = \frac{4}{a_0^3} r^2 e^{-\frac{2r}{a_0}} dr$ (ii) Show that the most probable distance of the

electron from the nucleus in the ground state of hydrogen is the Bohr radius a_o, (iii) Show that the average distance of the electron from the nucleus in the ground state of hydrogen is $3/2a_o$. [Hint: $\int_0^\infty z^n e^{-z} dz = n!$] (15%)

6. In the year of 1921, Stern and Gerlach used a beam of silver atom to pass through a non-uniform magnetic field and obtained the important results that demonstrated space quantization and furthermore, existing of electron spin. Describe in details, the Stern-Gerlach experiment set up, results and implications if a beam of hydrogen atom was used in stead of silver atom. (20%)