

國立清華大學命題紙

九十一學年度 電機工程、電子工程系(所) 丙 組碩士班研究生招生考試

科目 工程數學 科號 2601 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

1. Solve the following differential equations.

(a) $dy/dx = (2 \cos x - \sin x)\delta(x); y(-1) = 1$ (5%)

(b) $dy/dx = y \cosh x; y(0) = 1$ (5%)

2. Solve the following differential equations by LAPLACE TRANSFORM ONLY.

(a) $I(t) + \int_0^t I(t') dt' = 1$ (5%)

(b) $d^2y/dx^2 = -k^2y$ (Find the general solution.) (5%)

3. A function $f(x)$ is defined in the range $[-\pi, \pi]$ as follows: $f(x) = \begin{cases} 0 & \text{for } -\pi < x \leq 0 \\ 1 & \text{for } 0 < x \leq \pi \end{cases}$

Expand $f(x)$ into a complex Fourier series. (10%)

4. Find the Fourier series expansion for $f(t)$ and $|f(t)|$ with $f(t) = A \sin(\omega t + \phi)$, where A , ω , and ϕ are all positive constants. (10%)

5. Solve $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$, with $u(x=0, t) = u(x=3, t) = 0$ for all t , and $u(x, t=0) = \sin(14\pi x)$.

Derive a complete solution. (15%)

6. Solve $u_{xx} - 4u_{xy} + 3u_{yy} = 0$ by D'Alembert's method; that is, change independent variables and reduce the equation into a simplified form (the normal form), and then write down the general solution. (10%)

7. Evaluate $\int_0^{3+i} Z^2 dZ$ (a) along the line $y = \frac{x}{3}$; (b) along the real line to 3 and then

vertically to $3+i$. (10%)

國立清華大學命題紙

九十一學年度 電機工程、電子工程系(所) 丙 組碩士班研究生招生考試

科目 工程數學 科號 2601 共 2 頁第 2 頁 *請在試卷【答案卷】內作答

8. Conformal mapping (10%)

Find a linear fractional transformation that maps $|Z| \leq 1$ onto $|W| \leq 1$ such that $Z = \frac{i}{2}$ is mapped to $W=0$ and sketch the image of the lines $X=\text{constant}$ and $Y=\text{constant}$.

9. Laurent series expansion (15%)

Expand $f(z) = \frac{2z-1}{(z+1)(z-2)}$ into Laurent series centered at $z = -1$, i.e., into power

series in $z+1$. You should discuss the expansion in each regions of the complex plane and specify clearly the convergence region of each of your power series.