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九十一學年度 電機工程系 通訊工程研究所 乙甲 組碩士班研究生招生考試

科目 通訊系統 科號 2404 3002 共 3 頁第 1 頁 *請在試卷【答案卷】內作答

1. Consider a communication system where three equiprobable messages m_1, m_2, and m_3 are transmitted. Let m_1, m_2, and m_3 be encoded by signals $s_1(t)$, $s_2(t)$, and $s_3(t)$, respectively, given by

$$s_1(t) = 3\sqrt{2}\cos 2\pi t$$
$$s_2(t) = 2\sqrt{2}\sin 2\pi t$$
$$s_3(t) = -2\sqrt{2}\sin 2\pi t$$

where the signal duration is $0 \le t \le 1$ and each signal is zero outside this interval. Assume that the signals are transmitted over an additive white Gaussian noise channel.

(a) Find a set of orthonormal basis functions to represent the set of signals, and then draw the corresponding signal constellation. (5%)

(b) Determine the optimum decision regions. (5%)

(c) Determine an equivalent minimum-energy signal set that would yield the same probability of error as the signal set described above. Draw the corresponding signal constellation and optimum decision regions. (5%)

2 Consider a coherent binary frequency-shift keying (FSK) system where symbols 1 and 0 occur with equal probability. Let symbols 1 and 0 be encoded by signals $s_1(t)$ and $s_2(t)$, respectively, given by

$$s_i(t) = \begin{cases} \sqrt{2E_b / T_b}\cos(2\pi f_i t), & 0 \le t \le T_b \\ 0, & \text{elsewhere} \end{cases}$$

where i=1, 2, E_b is the transmitted signal energy per bit, T_b is the symbol duration, and $f_i=(n_c+i)/T_b$ for some fixed integer n_c. Assume that a white Gaussian noise process of zero mean and power spectral density $N_0/2$ is added during the transmission of an FSK signal.

(a) Determine the optimum receiver. (6%)

(b) Derive the error probability of the optimum receiver in terms of the complementary error function defined by

$$\text{erfc}(u) = \frac{2}{\sqrt{\pi}}\int_u^\infty \exp(-z^2)dz \,. \text{ (9\%)}$$

(You must give derivations, or you would get no points in this sub-problem!)

3. Consider the following random-phase sinusoidal process

$$x(t) = A\cos(\omega_0 t + \theta), -\infty < t < \infty$$

where ω_0 is a constant, θ is a random variable uniformly distributed over $[0,2\pi)$ and A is a binary random variable with probabilities $P_r[A=1]=p$ and $P_r[A=2]=1-p$.

(a) Find the mean and correlation function of $x(t)$. (10%)

(b) Hilbert transformer is a linear time-invariant system with frequency response

$$H(f)=\begin{cases} -j, & f>0 \\ j, & f<0 \end{cases}$$

Assume that $x(t)$ is input to the Hilbert transformer and $y(t)$ is the associated output. Find the power spectral density of $y(t)$. (5%)

4. Assume that

$$x(t) = as(t) + n(t), -\infty < t < \infty$$

where $n(t)$ is white Gaussian noise with zero mean and power spectral density $S_n(f)=1$, and the waveform of the signal $s(t)$ is given by

$$s(t)=\begin{cases} 1-t, & 0\le t\le 1 \\ 0, & \text{otherwise} \end{cases}$$

(a) Find the matched filter impulse response and peak output signal squared to output noise variance. (8%)

(b) Assume that $a=1$ or $a=-1$ with equal prior probability and that $y(t_0)$ is the matched filter output with the peak signal squared to output noise variance. Find the probability of error P_E of the detector that decides $a=1$ if $y(t_0)>0$ and $a=-1$ if $y(t_0)<0$. (You can express P_E in terms of the Q-function $Q(u)=\frac{1}{\sqrt{2\pi}}\int_u^\infty \exp(-z^2/2)dz$ or the complementary error function erfc(u)) (7%)

5. For a linear modulation with the in-phase component $s_I(t)=\frac{1}{2}m(t)$ and the quadrature component $s_Q(t)=\frac{1}{2}\hat{m}(t)$ ($\hat{m}(t)$ =Hilbert transform of $m(t)$), please show that $s(t)=s_I(t)\cos(2\pi f_c t)-s_Q(t)\sin(2\pi f_c t)$ is a single-sideband (SSB) with upper sideband transmitted signal. (10%)

6. Consider that a double sideband-suppressed carrier (DSB-SC) modulated signal $s(t) = A_c \cos(2\pi f_c t)m(t)$ is transmitted over an additive white Gaussian noise channel with power spectral density $N_0/2$, where $m(t)$ is the message signal with average message power P and message bandwidth W.
   (a) Find the average power of DSB-SC modulated signal $s(t)$ and the average noise power in the message bandwidth. (5 %)
   (b) Find the output signal of a coherent detector. (5 %)
   (c) Find the output signal-to-noise ratio. (5 %)

7. Assume that a linear time-invariant filter of impulse response $h(t)$ (frequency response $H(f)$) is driven by a stationary random process $X(t)$ (with power spectral density $S_X(f)$) and $Y(t)$ is the associated output.
   (a) Find the power spectral density $S_Y(f)$ of $Y(t)$. (10 %)
   (b) Show that the power spectral density $S_X(f)$ is always nonnegative. (5 %)