1. (15%) You are going to enhance a machine, and there are two possible improvements: either make multiply instructions run **four times faster** than before, or make memory access instructions run **two times faster** than before. You repeatedly run a program that takes 100 seconds to execute. Of this time, 20% is used for multiplication, 50% for memory access instructions, and 30% for other tasks.
   (a) What will the speedup be if you improve only multiplication?
   (b) What will the speedup be if you improve only the memory access?
   (c) What will the speedup be if both improvements are made?
   (d) If the program is changed so that the percentages are not the same but none of them is 0%, what sort of program will result in a tie with regard to speedup between the two individual improvements (you need to derive a formula for this computation)?
   (e) Amdahl's law is often written in terms of overall speedup as a function of two variables: the size of the enhancement (or amount of improvement) and the fraction of the original execution time that the enhanced feature is being used. Derive this form of equation from the two equations above.

2. (15%) A hypothetical single instruction computer SIC has only one instruction: subtract and branch if negative, or **sbn** for short. The sbn instruction has three operands, each consisting of the address of a word in memory:

   sbn a,b,c # Mem[a] = Mem[a] – Mem[b]; if (Mem[a]<0) goto c

   The instruction will subtract the number in memory location b from the number in memory location a and place the result back in a, overwriting the previous value. If the result is greater than or equal to 0, the computer will take its next instruction from memory location just after the current instruction. If the result is less than 0, the next instruction is taken from memory location c. SIC has no registers and no instructions other than sbn.
   Although it has only one instruction, SIC can imitate many of the operations of more complex instruction sets by using clever sequences of sbn instructions. For example, here is a program to copy a number from location a to location b:

```
start:   sbn   temp,temp,.+1     #sets temp to zero
         sbn   temp,a,.+1        #sets temp to –a
         sbn   b,b,.+1           #sets b to zero
         sbn   b,temp,.+1        #sets b to –temp which is a
```

In the program above, the notation ".+1" means "the address after this one," so that each instruction in this program goes on to the next in sequence whether or not the result is negative. We assume temp to be the address of a spare memory word that can be used for temporary results. Write a SIC program to add a and b, leaving the result in a and leaving b unmodified.

3. (10%) Let A = $a_{n-1}a_{n-2}\ldots a_1a_0$ be a two's complement integer. Show that

$$A = -a_{n-1}2^{n-1} + \sum_{i=0}^{n-2} a_i 2^i$$

4. (15%) Construct a 4-bit ALU by using 1-bit ALUs. You have to (a) build up a 1-bit ALU first by using AND, OR, and MUX gates, also use 1-bit adders as basic elements. (b) The 4-bit ALU has to support the following operation: add, subtract, AND, OR, set less than. (c) Also, there should be flags for zero and overflow.

5. (10%) Consider the comparisons of the horizontal and the vertical microinstructions for microprogramming. Answer the following two questions by giving your justifications.
   (a) Which format requires more control memory? Horizontal format or Vertical format?
   (b) Which format is more efficient to decode? Horizontal format or Vertical format?

6. (8%) A two-level virtual memory (M_1 and M_2) has access time $t_{a1} = 10^{-8}$ s and $t_{a2} = 10^{-3}$ s.
   (a) What must be the hit ratio H in order for the average access time t_a for the entire memory hierarchy to be 10^{-4} s.
   (b) Describe how to use the following two ways to reduce the memory access time t_a from 10^{-4} s to 10^{-5} s: (1) to increase hit ratio H or (2) decrease t_{a2}

7. (15%) Consider the following page address trace generated by a two-level cache-main memory scheme that use demand paging and a cache capacity of four pages

1 2 4 5 6 4 5 6 1 2 6 7 2 3 4 5 6 7 1 2 3 6 1 2 3 1 2

Assume a cold start in which the caches are initially empty. Which of the page replacement policies FIFO or LRU is more suitable (i.e., in terms of number of memory hits) in this case? Show your calculation by using the following table (for FIFO policy) to demonstrate the page address trace using the FIFO and LRU. You need two tables, one for each policy.

For FIFO policy

Time	1	2	3	4	5	6	7	8	9	10	11	12	.....
Page	1	2	4	5	6	4	5	6	1	2	6	7	.....
caches	0	0	0	1	2	2	2						
	0	0	1	2	4	4	4						
	0	1	2	4	5	5	5						
	1	2	4	5	6	6	6						
Hit						X	X						

8. (12%) Define each of the following three IO control methods: (a) Programmed IO, (b) DMA controllers, and (c) IOPs. List the advantages and disadvantages of each method with respect to (1) program complexity, (2) IO bandwidth, (3) interface hardware cost.