1. Short questions (15%)

(a) Write down the order of magnitude and unit of Planck constant, and list one experiment to measure it.

(b) Write down the definition of Hermitian operators. Explain why all quantum mechanical operators corresponding to physical measurements are required to be Hermitian.

(c) Explain the uncertainty principle and, starting from uncertainty principle, explain why identical particles are indistinguishable in quantum mechanics.

2. Infinite potential well band basic concepts (20%)

(a) A particle of mass m moves in a infinite potential well $V(x)=\begin{cases}0 & \text{for } |x|<a \\ \infty & \text{for } |x|>a\end{cases}$.

Starting from time-independent Schrodinger equation, find the eigenenergies E_n and normalized eigenstates $\phi_n(x)$.

(b) Continued to (a), supposed at t=0 the particle lies in a state $\Phi(x,t=0)=\frac{1}{2}\phi_2(x)+i\frac{\sqrt{3}}{2}\phi_3(x)$. Write down the total wavefunction, i.e., including the time dependence, of the particle at later time.

(c) Supposed now a particle lies initially in a mixed state described by a linear combination of the $\phi_n(x)$ described in (a). If the particle is subject to some perturbation and makes a transition to states with lower energy by emitting a photon of energy $5E_1$, where E_1 is the ground-state energy. Can one conclude from this observation that the particle lies initially in $\phi_3(x)$? Explain why.

3. The eigenfunction $\phi(x) = c\ x \exp(-\alpha x^2)$ describes a state of quantum harmonic oscillator, provided the constant α is chosen appropriately.

(a) Find α and the energy of this state in terms of classical frequency of vibration ω.

(b) Is this state the ground state, 1st excited state, 2nd excited state or 3rd excited state.....etc. ?

(Hint: The potential energy of a classical harmonic oscillator is $(1/2)m\omega^2x^2$.) (10%)

4. The following phenomena are due to tunneling effect: (a) Field emission from metal surface, (b) Leaky current of a parallel plate capacitor made of ideal insulator, (c) Alpha decay, (d) Ammonia molecule inversion. Pick <u>any</u> one of the above choices and describe in detail how it does happen. (10%)

5. A quantum free particle can be described by the wavefunction $\Psi(x,t)=A\exp[i(kx-\omega t)]$. What can you say about the position of this particle? What about the momentum of this particle? What is the energy of this particle? How much time is needed in order to measure the energy of this particle? (15%)

6. Consider a particle with mass = m, moving in the 3-dimensional potential

$$V(\vec{r})=\frac{1}{2}k|\vec{r}|^2=\frac{1}{2}kr^2$$, where k is a constant.

Which of the following statements are true? (15%) (複選)

(a) The energy eigenstates of the particle can be written as $R_n(r)Y_{lm}(\theta,\phi)$, that is, the radial and angular dependences can be separated, where (r,θ,ϕ) designates the spherical coordinates.

(b) $R_n(r)$ satisfies the equation $-\frac{\hbar^2}{2m}\frac{\partial^2 R_n}{\partial r^2}+V(\vec{r})R_n=E_nR_n$.

(c) The lowest energy eigenstate has a spherically symmetric distribution of probability.

(d) If we increase the value of k, the lowest allowed energy will also increase.

(e) All of the above are correct.

7. Consider the system of two identical, spinless, independent Fermi particles moving in a one-dimensional infinite potential well, maintained at temperature T. Let E_n be the energy levels of the well, with n = 0,1,2……, and E_{total} be total energy of the two particles. Which of the following statements are true? (15%) (複選)

(a) Each particle has an energy which obeys the Fermi-Dirac distribution.

(b) When T is finite, the system may have various energies, with the following probability distribution, $P(E_{total}=E) \propto \exp(-E/k_BT)$.

(c) At $T=0$, $E_{total}=2E_0$.

(d) Let's arrange the various total energies in an increasing order, that is, $E_{ground} < E_{1st\ excited} < E_{2nd\ excited} < \ldots\ldots$ Then, $E_{2nd\ excited}-E_{ground}=E_2-E_0$.

(e) All of the above are correct.