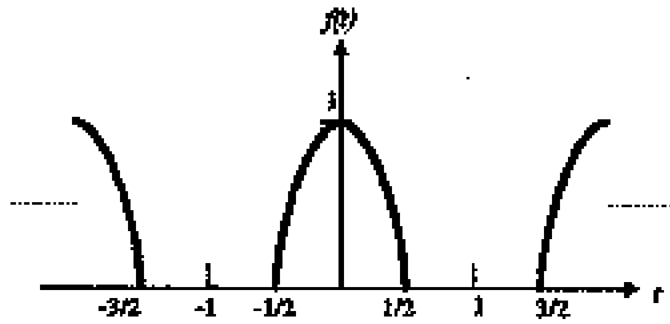
1. Consider a causal and stable linear time-invariant system

$$y(n)-\frac{1}{12}y(n-1)-\frac{1}{12}y(n-2)=x(n)$$

Please determine its frequency response and impulse response. (10%)

2. Determine the Fourier series expansion for an output signal from the half-wave rectifier. (10%)



3. Find the inverse Fourier transform of the function

$$F(\omega)=\frac{2a}{a^2+\omega^2}$$

(You should consider the cases of t > 0 and t < 0 separately.) (10%)

4. (a) Show that an LTI system with impulse response $h[n]$ is BIBO-stable, if $h[n]$ is absolutely summable. (6%)

(a) If, on the contrary, $h[n]$ is not absolutely summable,

(i) Suppose that the input to the system is

$$x[n]=\begin{cases}0 & \text{if } h[-n]=0\\ \dfrac{h^*[-n]}{|h[-n]|} & \text{if } h[-n]\neq 0\end{cases}$$

Does the input signal represent a bounded input ? (1%)

If so, what is the smallest number B such that $|x[n]|\leq B$ for all n ? (1%)

(ii) Calculate the output at $n=0$ for the above particular input. (3%)

How do you interpret this result ? (4%)

5. For a low-pass filter, the desired response is

$$H_d(e^{j\omega})=\begin{cases}e^{-j\omega\tau} & |\omega|\leq\omega_c<\pi\\ 0 & otherwise\end{cases}$$

(a) Find the filter coefficient $h_d[n]$ for $n\neq\tau$ and $n=\tau$. (6%)

(b) Determine τ so that $h_d[n]=h_d[N-1-n]$. (4%)

6. Suppose a causal LTI system S with input $x[n]$ and system function H(z) given as

$$H(z)=\frac{1-\frac{5}{4}z^{-1}+\frac{3}{8}z^{-2}}{(1+\frac{1}{4}z^{-1}-\frac{1}{8}z^{-2})}.$$ The system H(z) can be cascaded of two subsystems as

H(z)=H1(z)H2(z), where $H1(z)=\frac{1}{(1+\frac{1}{4}z^{-1}-\frac{1}{8}z^{-2})}$ and H2(z)= $1-\frac{5}{4}z^{-1}+\frac{3}{8}z^{-2}$

(a) Draw the block diagram of H(z) as a cascade connection of a block diagram of H1(z) followed by a block diagram of H2(z) using four unit delay elements which are called the direct form I.

(b) Modify the direct form I to direct form II which requires only two unit delay elements.

(c) Draw the block diagram of H(z) in parallel form which may only need two unit delay elements. (15%)

7. The z-transform of $x[n]$ is given as $X(z)=\frac{1-\frac{1}{4}z^{-1}}{(1+z^{-1}-2z^{-2})}$ (15%)

(a) Can $x[n]$ be a finite duration signal, why ?

(b) If $x[n]$ is two-sided sequence, then where is the region of convergence ? Is it stable ?

(c) If $x[n]$ is right-sided sequence, then where is the region of convergence ? Is it stable ?

(d) If $x[n]$ is left-sided sequence, then where is the region of convergence ? Is it stable ?

8. The sequence $x(t)$ can be expanded in a Taylor series at t=0+ as

$$x(t)=[x(0+)+x^{(1)}(0+)t+\ldots+x^{(n)}(0+)t^n/n!+\ldots]u(t).$$ (15%)

Now, please answer the following questions:

(a) Use the property of the Laplace transform of $e^{-at}(t^n/n!)u(t)$ is $1/(s+a)^n$ to find X(s) in terms of $x(0+), x^{(1)}(0+), \ldots\ldots, x^{(n)}(0+)\ldots$(e.g. let a=0)

(b) Use the above property to prove the initial value theorem

$$x(0+)=\lim_{s\to\infty} sX(s)$$

(c) Use the initial value theorem to find $x(0+)$ given X(s) = s/(s+2)(s+4)