1. For the message signal $m(t) = \frac{1}{1+t^2}$, determine and sketch the modulated waves for the following methods of modulation:

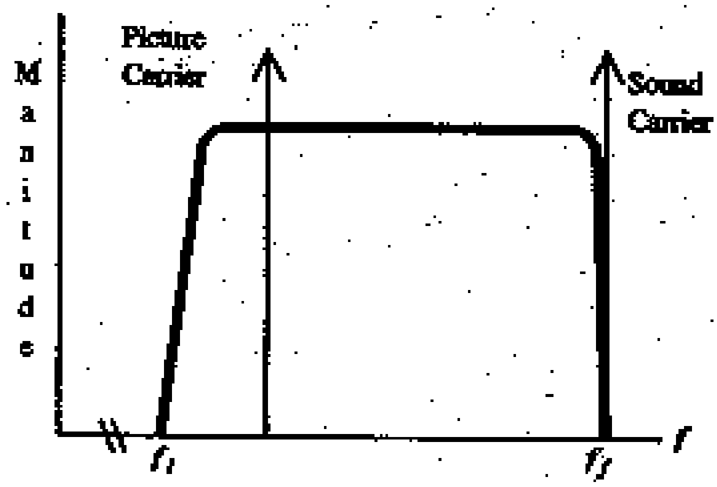
   (a) (5%) Amplitude modulation with modulation index $k_a = 1$.

   (b) (5%) Double sideband-suppressed carrier modulation.

   (c) (5%) Single sideband modulation with only the upper sideband transmitted.

   (d) (5%) Single sideband modulation with only the lower sideband transmitted.

2. An idealized magnitude spectrum of a transmitted TV signal is shown below.



   (a) (3%) Explain why the TV signal using VSB modulation not DSB or SSB modulation.

   (b) (3%) Where is the location of VSB filter? (transmitter or receiver) Why?

   (c) (2%) What is the role of 'Picture Carrier'?

   (d) (2%) What kind of demodulation in the receiver is used?

3. Consider a narrowband FM signal approximately defined by

$$s(t) \cong A_c\cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t)\sin(2\pi f_m t) \tag{1}$$

   (a) (3%) Determine the envelope of this modulated signal. What is the ratio of the maximum to the minimum value of this envelope?

   (b) (3%) Determine the average power of this narrowband FM signal and express as a percentage of the average power of the unmodulated carrier wave.

   (c) (4%) Please show that the angle of the narrowband FM signal

$$\theta_i(t) \cong 2\pi f_c t + \beta\sin(2\pi f_m t) - \frac{\beta^3}{3}\sin^3(2\pi f_m t) \tag{2}$$

   [Hint: $\tan \cong x - \frac{x^3}{3} + \ldots$]

4. (10%) Let $s(t) = m(t)c(t)$ where $m(t)$ is a real low-pass signal whose Fourier transform $M(f)$ has $M(f) = 0$ for all $|f| \geq W$ for some $W > 0$ and $c(t)$ is a real high pass signal whose Fourier transform $C(f)$ has $C(f) = 0$ for all $|f| \leq W$. Please show that the Hilbert transform $\tilde{s}(t)$ of $s(t)$ is equal to $m(t)\tilde{c}(t)$ with $\tilde{c}(t)$ the Hilbert transform of $c(t)$.

5. Consider a binary frequency-shift keying (BFSK) digital modulated signal transmitted with a carrier whose phase is unknown at the receiver, i.e., the BFSK becomes noncoherent at the receiver. The two possible transmitted signals can be modeled as

$$s_0(t) = \cos(2\pi f_0 t + \theta_0) \text{ and } s_1(t) = \cos(2\pi f_1 t + \theta_1)$$

for $t \in [0, T_b]$ to represent 0 and 1 respectively, where θ_0 and θ_1 are two unknown phases.

   (a) (5%) What is the minimum possible separation $|f_0 - f_1|$ of f_0 and f_1 such that $s_0(t)$ and $s_1(t)$ are orthogonal, i.e., $\int_0^{T_b} s_0(t)s_1(t)dt = 0$, no matter what θ_0 and θ_1 are? Please explain your answer, otherwise no credits.

   (b) (5%) How can we exploit the orthogonality between $s_0(t)$ and $s_1(t)$ to design an optimal receiver for this noncoherent BFSK?

6. (10%) A radio link is modeled as a two-path communication channel with impulse response

$$h(t) = \delta(t) + 0.9 \cdot \delta(t - \Delta)$$

where $\delta(t)$ is the delta function and $\Delta = 0.2$ μsec is the delay of the second path relative to the first one. If you want to set up a binary phase shift keying (BPSK) modulation system for this radio link, what is the possible range of transmission bit rates without severe bit error rate? And, what is the possible carrier frequency to be used? You should explain your answer, otherwise no credits.

7. (15%) Consider a direct-sequence spread-spectrum system with BPSK modulation. After transmitted signal going through an additive-white-Gaussian-noise channel, we measured both the ship rate and the average signal-to-noise ratio just before de-spreading and obtained 2×10^6 Hz and -10 dB, respectively. We wish a target bit error rate of 10^{-6}, determine the maximum throughput of the system in terms of Q function, where Q function is defined as $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp(-\frac{x^2}{2})dx$.

8. Consider a scalar digital communication system $y = x + n$, where n is an zero-mean additive Gaussian noise with variance σ^2. Two possible value of x is transmitted, $x = x_0$ with probability of p_0 and $x = x_1$ with probability of p_1.

   (a) (10%) Determine the optimum receiver in terms of x_0, x_1, p_0, p_1 and σ.

   (b) (5%) Determine the average error probability of the optimum receiver obtained in (a). Please express your answer in term of Q function, where Q function is defined as $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp(-\frac{x^2}{2})dx$.