1. Solve for y as a function of x from the differential equation
$(2\cosh y+3x)dx+(x\sinh y)dy=0$. (6%)

2. Find the complete solution for y, where y satisfies the differential equation
$y''-4y'+4y=2e^{2x}$ and the initial conditions $y(0)=3$, $y'(0)=4$. (6%)

3. Use the Laplace transform to solve y_1 and y_2 from the nonhomogeneous linear differential equation system $\begin{bmatrix} y_1' \\ y_2' \end{bmatrix}=\begin{bmatrix} -3 & 1 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}+\begin{bmatrix} -6 \\ 2 \end{bmatrix}e^{-2t}$, with their initial conditions given by $y_1(0)=y_2(0)=0$. (8%)

4. Solve the partial differential equation

$$\frac{\partial u}{\partial x}=\frac{\partial^2 u}{\partial t^2},$$

where u(x,t) satisfies the following requirements
$u(x=0,t)=100\pi\cdot\sin(100\pi t)$ for all $0\leq t<1$,
$u(x,t=0)=u(x,t=1)=0$ for all $x\geq 0$. (12%)

5. (a) Express $\cos(\frac{z}{z-1})$ into a power series for some region of the complex plane. (6%)

(b) Compute the following complex integrals

$$\int_{C1}\cos(\frac{z}{z-1})dz \quad \text{and} \quad \int_{C2}\cos(\frac{z}{z-1})dz$$

where C1 and C2 are counterclockwise contours (circles) with center z=0 and radii of 0.5 and 2,respectively. (12%)

6. Let $f(t)=t-[t]$, where $[t]$ denotes the largest of the integers which are smaller than t.

(a) Find the Laplace transform of $f(t)$. (7%)

(b) Since $f(t)$ is periodic, it may be expaned in Fourier series, i.e., in the form $\frac{a_0}{2}+\sum_{n=1}^{\infty}\{a_n\cos(n\omega t)+b_n\sin(n\omega t)\}$. Calculate and write down ω, a_0, a_1 and b_1. (8%)

(c) Solve $\frac{d^2y}{dt^2}+2\frac{dy}{dt}+2y=f(t)$ with the initial conditions $y(0)=1, y'(0)=1$ (5%)

7. (a)Evaluate the line integral $\int_{(0,2)}^{(1,\pi)}\sin xy\,(ydx+xdy)$. (10%)

(b)Evaluate the double integral $\iint_R f(x,y)dxdy$,

where $f(x,y)=\cos(x^2+y^2)$, $R: x^2+y^2\le\pi/2$, $x\ge 0$. (10%)

8. Consider a $n\times n$ real-valued matrix A. Which of the following statements are equivalent to "A is nonsingular"? (10%) (Proofs are not needed. Simply choose the equivalent statements. No partial credit for this problem.)

(a) A is invertible.

(b) $A\mathbf{x}=\mathbf{0}$ has a solution $\mathbf{0}$.

(c) The system of n linear equations in n unknowns $A\mathbf{x}=\mathbf{e}_1$ has a unique solution, where $\mathbf{e}_1=(1,0,\ldots,0)^T$.

(d) A^2+3A+I is nonsingular.

(e) A^2+4A is nonsingular.

(f) The column vectors of A are linearly independent.

(g) The row vectors of A spans $\mathcal{R}^n$.

(h) A is similar to some matrix C.

(i) A is a transition matrix with respect to some ordered basis to the standard basis.

(j) A is a matrix representing some linear transformation.