1. Consider a filter with frequency response given by

$$H(j\omega)=\frac{2(10^3)}{\omega^2+10^6}e^{-j3\omega}$$

(a) Find the impulse response of the filter. (8%)

(b) Is the filter physically realizable? You must justify your answer, or you will get no points. (4%)

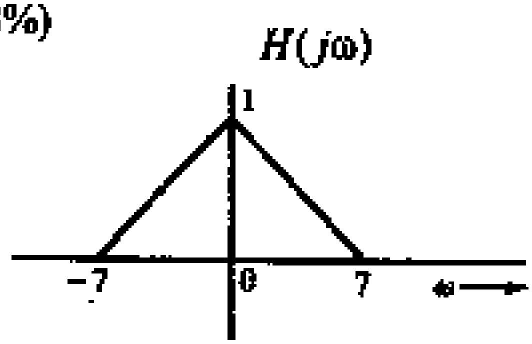
2. Consider a system with frequency response $H(j\omega)$ shown below.

(a) Find the energy of the impulse response of the system. (5%)

(b) Determine the output of the system when the input is given by $x(t)=2(\sin^2 t)(\cos 6t)$. (5%)

(c) Determine the output of the system when the input is an impulse train given by

$x(t)=\sum_{k=-\infty}^{\infty}\delta(t-2k-1)$. (8%)



3 For the first-order causal LSI system described by the difference equation

$$y[n]-ay[n-1]=x[n],\ \text{with}\ |a|<1.$$

(a) Find the impulse response and plot it, (3%)

(b) Find the step response and plot it, (3%)

(c) Find the frequency response and plot its magnitude and phase response. (6%)

For the plotting, a>0 and a<0 should be treated separately.

4 For a discrete system described by

$$H(z)=\frac{1-2z^{-1}}{1-\frac{1}{2}z^{-1}},\quad |z|>\frac{1}{2}$$

(a) Is it stable? Is it causal? (2%)

(b) Find the inverse system which is causal but not stable. (3%)

(c) Find the inverse system which is stable but not causal. (3%)

5 Sampling can be modeled as the multiplication of the input signal $x(t)$ by the uniform impulse train

$$p(t)=\sum_{k=-\infty}^{\infty}\delta(t-kT)$$

where $T=\frac{2\pi}{\omega_s}$ is the sampling period and ω_s is the sampling frequency.

(a) Derive the sampled spectrum in terms of the spectrum $X(j\omega)$ of the input signal. (8%)

(b) From the above result, induce the sampling theorem. (2%)

(c) What is the physical interpretation of the sampling theorem on a pure sinusoidal signal? (2%)

(d) Suppose that the sampling theorem is satisfied, show that the original spectrum can be recovered via an ideal low-pass filter and derive the recovered signal in the t-domain. (8%)

6. When the impulse train (10%)

$$x[n] = \sum_{k=-\infty}^{\infty} \delta[n-4k]$$

is the input the a particular LTI system with frequency response $H(e^{j\omega})$, the output of the system is found to be

$$y[n]=\cos((5\pi/2)n+(\pi/4))$$

Determine the values of $H(e^{jk\pi/2})$ for k=0, 1, 2 and 3.

7. Consider a signal x(t) with Fourier transform X(jω). Suppose we are given the following facts: (10%)

(a) $x(t)$ is real and nonnegative.

(b) $F^{-1}\{(1+j\omega)X(j\omega)\}=Ae^{-2t}u(t)$, where A is independent of t.

(c) $\int_{-\infty}^{\infty}|X(j\omega)|^2 d\omega = 2\pi$.

Determine a close-form expression for $x(t)$.

8. The Fourier transform of a particular signal is (10%)

$$a^{|n|} \longrightarrow \frac{1-a^2}{1-2a\cos\omega+a^2} \qquad |a|<1$$

use duality to determine the Fourier series of the following continuous time signal with period T=1

$$x(t)=1/[5-4\cos(2\pi t)]$$