國 立 清 華 大 學 命 題 紙

八十八學年度 電机工程 系（所） 丙 組碩士班研究生招生考試

科目 近代物理 科號 4604 共 2 頁第 1 頁 *請在試卷【答案卷】內作答

1. In the classical limit, Newton's equation of motion is more convenient than the Schrodinger equation for describing the dynamics of moving particles. The connection between the Newtonian and quantum mechanics is established by the Ehrenfest theorem: $\frac{d<x>}{dt}=\frac{1}{m}<p_x>$ and $\frac{d<p_x>}{dt}=-<\frac{\partial V}{\partial x}>$. Starting from the definition of expectation value and Schrodinger equation, prove the Ehrenfest theorem. (15%)

2. A magnetic moment is associated with the circular motion of a charged particle. Let the charge = e, mass = m, velocity = v, and radius = r.
   (A) Starting the definition of magnetic moment $\vec{\mu}=IA\hat{a}$, where I is the current, A is the area of the circular orbital, and $\hat{a}$ is the unit vector directing along the axis of the circular orbit. Express the magnetic moment in term of e, m, v, and r. (5%)
   (B) In the presence of a magnetic field (0, 0, B) along the z-direction, the part of the Hamiltonian operator associated with the magnetic moment is $H_B=-\vec{\mu}\bullet\vec{B}$ and can be expressed as $H_B=CL_zB$, where L_z is the z-component of the angular momentum operator. Please determine the constant C. (5%)
   (C) Supposed that the charged particle is spinless, and is moving in a hydrogen-1p-like orbital. This charged particle may fall into a hydrogen-1s-like orbital and emits photon. If now a magnetic field (0, 0, B) is turned on, determine the number of line spectra and the energy splitting between these lines which would be observed. (10%)

3. A particle of mass m moves one dimensionally in a potential $(x)=-A\delta(x)$, where A is a positive constant. Starting from time-independent Schrodinger equation, write down the conditions that the bound-state wavefunction would satisfy at the origin and solve for the bound-state energies. (Hint: $V(x)=V(-x)$) (15%)

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4. (A). Explain what exchange energy is and why the exchange energy is always negative for electrons. (10%)

   (B). Consider only one-dimensional bound state problems. Is it true that the bound-state wavefunction can always be taken as a real function if the potential V(x) is real? Explain in details your reasoning. (10%)

5. A particle is trapped in an infinitely deep one-dimensional box with a width of L. Calculate the expectation value of x. The following integral is given.

   $$\int x\sin^2 x\,dx = \frac{x^2}{4} - \frac{x\sin 2x}{4} - \frac{\cos 2x}{8}$$ (15%)

6. Find the average kinetic energy per electron of a three-dimensional electron gas at zero degree temperature (0K). It is known that the density of states of free electrons is proportional to the square root of energy. (15%)