

八十八學年度 電機工程 系(所) 丙 組碩士班研究生招生考試

科目 工程數學 科號 4601 共 2 頁第 1 頁 請在試卷【答案卷】內作答

1. (A) Find the general solution for $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = e^{-x} \cos(2x+1)$. (5%)

(B) $f(x) = \begin{cases} 1 & \text{for } 0 \leq x < 1 \\ 0 & \text{for } 1 \leq x < 2 \end{cases}$ and $f(x+2) = f(x)$, find the Laplace transform

of $f(x)$ and solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = f(x)$, $y(0) = 0$, $y'(0) = 0$. (10%)

2. (A) From the properties of Dirac delta function, expand $\delta(1-4t^2)$ as the sum of delta functions with simple argument; that is, find the parameters A_n and a_n such that $\delta(1-4t^2) = A_1\delta(t-a_1) + A_2\delta(t-a_2) + \dots$ holds. (5%)

(B) Solve the following equation by Laplace transform:

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 2y = \delta(1-4t^2), \quad y(0) = 0, \quad y'(0) = 0. \quad (5\%)$$

(C) Is the solution $y(t)$ in (B) continuous at $t = \frac{1}{2}$? If not, how are $y(t^+)$ and $y(t^-)$ related? Explain how you can figure out this relationship simply from the equation itself without actually solving for the solution. (5%)

3. Let P_3 be the set of all polynomials with degree less than 3.

(a) (5%) Find the transition matrix S from the ordered basis $[1, x, x^2]$ to the ordered basis $[1, 2x, 4x^2 - 2]$.

(b) (5%) Let D be the differentiation operator on P_3 . Find the matrix A representing D with respect to the basis $[1, 2x, 4x^2 - 2]$.

4. Show the following Fourier transform theorems: (12%)

(a) convolution theorem $F\{f * g\} = \sqrt{2\pi}F\{f\}F\{g\}$.

(b) shifting theorem: $F\{f(x-a)\} = e^{-j\omega a}F\{f(x)\}$.

(c) autocorrelation theorem: $F\{\int f(\xi)f(\xi-x)d\xi\} = \sqrt{2\pi}|F\{f\}|^2$.

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5. Evaluate the following integral: (10%)

$$\int_0^{\infty} \frac{x - \sin x}{x^3(x^2 + a^2)} dx, a > 0.$$

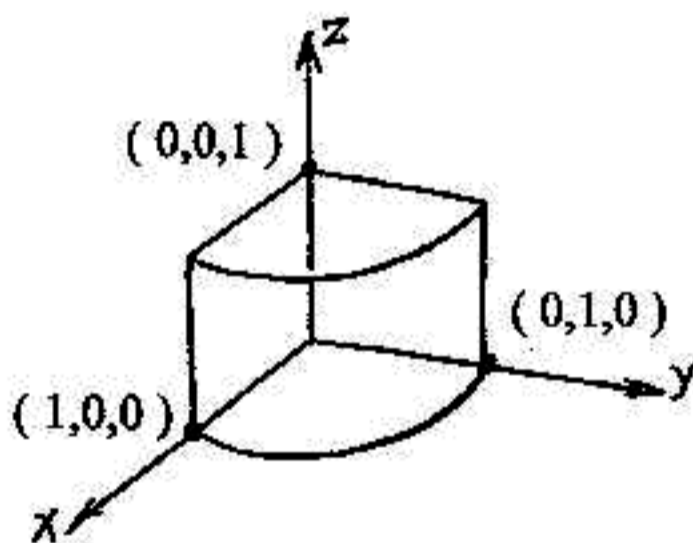
6. Show that if $a > 0$,

$$\int_0^{\infty} \frac{x - \sin x}{x^3(a^2 + x^2)} dx = \frac{\pi}{2a^4} \left(\frac{a^2}{2} - a + 1 - e^{-a} \right). \quad (10\%)$$

7. (a) (4%) Find a vector field $\vec{F}(x,y,z)$ such that $\nabla \cdot \vec{F} = x^3 + y$.

(b) (11%) Compute the integral $\int_V (x^3 + y) dx dy dz$

using the divergence theorem, where V represents a quarter of the cylinder as shown below.



8. Solve the partial differential equation

$$\frac{\partial^3 u}{\partial t^3} = \frac{\partial u}{\partial x}$$

where u is a function of t and x satisfying $u(t, x = -\infty) = 0$, $u(t = 0, x) = 0$,

$$\frac{\partial u}{\partial t} \Big|_{t=0} = 0 \quad \text{and} \quad \frac{\partial^2 u}{\partial t^2} \Big|_{t=0} = e^{ax}. \quad (13\%)$$