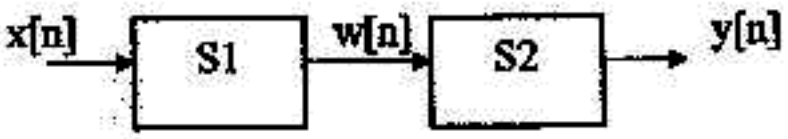
1. (15%)

(a) Show that if a signal $x(t)$ is causal and contains no impluse (or higher-order singularity) at t=0, the initial value $x(0^+)$ is given by the following Laplace transform limit: $x(0^+) = \lim_{s\to\infty} sX(s)$.

(b) IF $X(s)=s/(s^2+4)$, then determine the initial value of $x(t)$.

2. (15%)

Consider the cascade of the following two systems S1 and S2, depicted as



S1: causal LTI: $w[n]=\alpha w[n-1]+\beta x[n]$.

S2: causal LTI: $y[n]=(1/2)y[n-1]+w[n]$.

The difference equation relating x[n] and y[n] is

$$y[n]=(-1/4)y[n-2]+y[n-1]+x[n].$$

(a) Determine α and β.

(b) Show the impulse response of the cascade connection of S1 and S2.

3. (15%)

Consider a causal LTI system S whose input x[n] and output y[n] are related by the difference equation

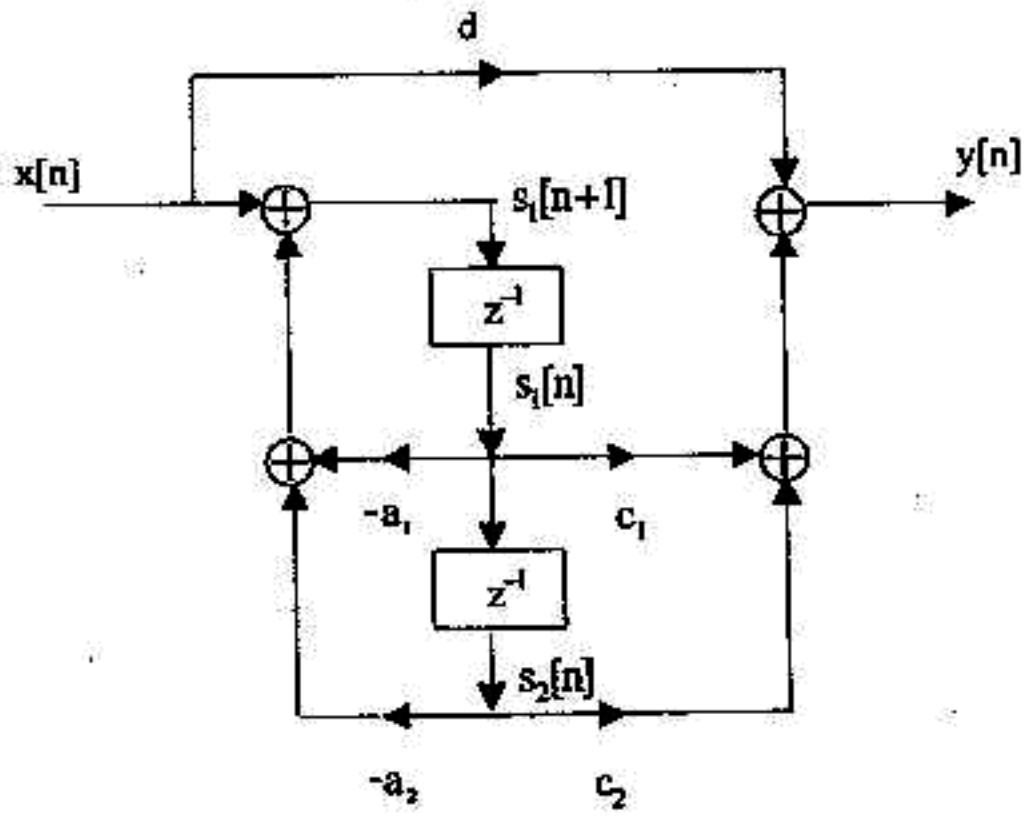
$$ay[n] + by[n-1] = cx[n]+dx[n-1]$$

S may be considered as a cascade connection of two causal LTI system S1 and S2 using the unit-delay element.

(a) Draw a block diagram representation of S as a cascade connection of two causal LTI systems using two unit-delay elements. (direct Form I)

(b) Draw another diagram representation of S to show that the two-delay elements in the above diagram may collapse into one delay element. (Direct From II)

4. In the following second-order discrete-time network with states $s_1[n]$ and $s_2[n]$, (15%)



Please find the state equation

$$S[n+1]=AS[n]+Bx(n)$$
$$y[n]=CS[n]+Dx(n)$$

Where $S[n]=\begin{bmatrix} s_1[n] \\ s_2[n] \end{bmatrix}$, i.e., find the system matrices A, B, C and D.

5. Given the accumulator system function (15%)

$$H(z)=\frac{1}{1-z^{-1}},\ |z|>1$$

Please find (i) the impulse response of H(z) (ii) the impulse response of the associated inverse system (iii) Please give a first-order linear difference equation with input x[n] and output y[n] to describe the accumulator H(z).

6. Consider an ideal lowpass filter with impulse response $h(t)=\frac{\sin(1.5t)}{\pi t}$.

(a) Determine and plot the frequency response of the filter. (5%)

(b) Determine the output of the filter when the input is $x(t)=\cos t+\sin^2 t$. (5%)

(c) Determine the output of the filter when the input is $x(t)=|\sin t|$. (5%)

7. Consider the following two finite-duration sequences:

$$x_1[n]=u[n-2]-u[n-7]$$

$$x_2[n]=u[n]-u[n-5]$$

where $u[n]$ is the unit-step sequence. Assume that we zero-pad $x_1[n]$ and $x_2[n]$ appropriately to form two new sequences $x'_1[n]$ and $x'_2[n]$ with the same length of K points.

(a) Determine the minimum value of K such that the circular convolution of $x'_1[n]$ and $x'_2[n]$ can generate all points of the linear convolution of $x_1[n]$ and $x_2[n]$. (3%)

(b) If K=8, specify which points of the circular convolution of $x'_1[n]$ and $x'_2[n]$ are corresponding to points that would be obtained in the linear convolution of $x_1[n]$ and $x_2[n]$. (7%)